Integration Worksheet 1 - Understanding the Definite Integral

Show all work on your paper as described in class. Video links are included throughout for instruction on how to do the various types of problems. **Important:** Work the problems to match everything that was shown in the videos. For example: Suppose a video shows 3 ways to do a problem, (such as algebraically, graphically, and numerically), then your work should show these 3 ways also. That is, each video is a model for the work I want to see on your paper.

1-R: Read and take notes on Section 5.1

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

**Estimate the value of the quantity.**

1) The table shows the velocity of a remote controlled race car moving along a dirt path for 8 seconds. Estimate the distance traveled by the car using 8 sub-intervals of length 1 with left-end point values.

http://youtu.be/UJBEAXpjZVI

(The geeks among you will get a kick out of the Lagrange Interpolating Polynomial that can be used to fit a given set of points with a polynomial (which could then be graphed as be a possible function for the data). Go to this site: http://www.solvemymath.com/online_math_calculator/interpolation.php, how it works is shown at http://www.wolframalpha.com/input/?i=lagrange+polynomial&lk=4 )

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Velocity (in./sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
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<tr>
<td>4</td>
<td>22</td>
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<td>5</td>
<td>25</td>
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<td>6</td>
<td>27</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

A) 114 in.  
B) 124 in.  
C) 248 in.  
D) 129 in.

2) The table shows the velocity of a remote controlled race car moving along a dirt path for 8 seconds. Estimate the distance traveled by the car using 8 sub-intervals of length 1 with right-end point values.

http://youtu.be/8iEQQD897bs

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Velocity (in./sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
</tr>
<tr>
<td>5</td>
<td>27</td>
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<tr>
<td>6</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>25</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

A) 176 in.  
B) 166 in.  
C) 172 in.  
D) 182 in.
3) Joe wants to find out how far it is across the lake. His boat has a speedometer but no odometer. The table shows the boat's velocity at 10 second intervals. Estimate the distance across the lake using right-end point values.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Velocity (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
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<tr>
<td>30</td>
<td>53</td>
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<tr>
<td>40</td>
<td>50</td>
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<td>50</td>
<td>55</td>
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<tr>
<td>60</td>
<td>52</td>
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<tr>
<td>70</td>
<td>55</td>
</tr>
<tr>
<td>80</td>
<td>45</td>
</tr>
<tr>
<td>90</td>
<td>15</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

A) 3670 ft  B) 367 ft  C) 3770 ft  D) 5500 ft

4) A piece of tissue paper is picked up in gusty wind. The table shows the velocity of the paper at 2 second intervals. Estimate the distance the paper travelled using left-endpoints.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Velocity (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>10</td>
<td>26</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
</tr>
</tbody>
</table>

A) 182 ft  B) 202 ft  C) 101 ft  D) 179 ft

5) The velocity of a projectile fired straight into the air is given every half second. Use right endpoints to estimate the distance the projectile travelled in four seconds.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Velocity (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>139</td>
</tr>
<tr>
<td>0.5</td>
<td>134.1</td>
</tr>
<tr>
<td>1.0</td>
<td>129.2</td>
</tr>
<tr>
<td>1.5</td>
<td>124.3</td>
</tr>
<tr>
<td>2.0</td>
<td>119.4</td>
</tr>
<tr>
<td>2.5</td>
<td>114.5</td>
</tr>
<tr>
<td>3.0</td>
<td>109.6</td>
</tr>
<tr>
<td>3.5</td>
<td>104.7</td>
</tr>
<tr>
<td>4.0</td>
<td>99.8</td>
</tr>
</tbody>
</table>

A) 935.6 m  B) 974.8 m  C) 487.4 m  D) 467.8 m
In the following problems, \( f(x) \) represents the velocity of an object in ft/sec at time \( x \) seconds moving on a straight track. Estimate the distance the object has traveled during the given time interval using the given number of rectangles and using left or right sums as instructed.

6) \( f(x) = x^2 \) between \( x = 0 \) and \( x = 4 \) using a left sum with two rectangles of equal width.

\[ \text{http://youtu.be/2NB-KJT2sMg} \]

A) 8  B) 38.75  C) 20  D) 40

7) \( f(x) = x^2 \) between \( x = 0 \) and \( x = 1 \) using a right sum with two rectangles of equal width.

\[ \text{http://youtu.be/6httzrxmzoA} \]

A) .625  B) .75  C) .125  D) .3145

8) \( f(x) = x^2 \) between \( x = 2 \) and \( x = 6 \) using a left sum with four rectangles of equal width.

A) 86  B) 54  C) 69  D) 62

9) \( f(x) = \frac{1}{x} \) between \( x = 1 \) and \( x = 9 \) using a right sum with two rectangles of equal width.

\[ \begin{array}{c|c|c|c}
   & \frac{56}{45} & \frac{24}{5} & \frac{8}{15} & \frac{56}{5} \\
\hline
A) & & & & \\
   B) & & & & \\
   C) & & & & \\
   D) & & & & \\
\end{array} \]

10) \( f(x) = x^2 \) between \( x = 1 \) and \( x = 5 \) using a right sum with four rectangles of equal width.

A) 69  B) 41  C) 54  D) 30

11) \( f(x) = \frac{1}{x} \) between \( x = 1 \) and \( x = 6 \) using an left sum with two rectangles of equal width.

\[ \text{http://youtu.be/xMpERZLuh0o} \]

\[ \begin{array}{c|c|c|c|c}
   & \frac{95}{14} & \frac{15}{28} & \frac{45}{14} & \frac{95}{84} \\
\hline
A) & & & & \\
   B) & & & & \\
   C) & & & & \\
   D) & & & & \\
\end{array} \]

ESSAY.

12) Write these notes on your paper:

The rectangle sums we have been computing, above, are called \textbf{Riemann sums} (pronounced "Ree–mon"),

\[ \sum f(x) \Delta x \]  where \( x \) is the left endpoint of a sub interval for a "\textbf{left Riemann sum}" and \( x \) is the right endpoint for a "\textbf{right Riemann sum}". And \( \Delta x = \frac{\text{right endpoint of interval} - \text{left endpoint of interval}}{\text{number of sub intervals}} = \frac{b - a}{n} \)

(For the above problems and these below, you can verify your answers using the following Riemann sum applet: \text{http://mathworld.wolfram.com/RiemannSum.html} )

13) Go to \text{http://en.wikipedia.org/wiki/Bernhard_Riemann} and write a brief paragraph report on \textbf{Bernhard Riemann}
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Graph the function $f(x)$ over the given interval. Partition the interval into 4 sub-intervals of equal length. Then add to your sketch the rectangles associated with the left or right Riemann sums as directed.

14) $f(x) = 2x + 4$, $[0, 2]$, left-hand endpoint  See previous videos where the rectangle concept is explained
15) \( f(x) = -2x - 1, [0, 2], \) left-hand endpoint

16) \( f(x) = x^2 - 2, [0, 8], \) left-hand endpoint
17) \( f(x) = x^2 - 2, \ [0, 8], \) right-hand endpoint
18) $f(x) = -3x^2$, $[0, 4]$, left-hand endpoint
19) \( f(x) = \cos x + 2, \ [0, 2\pi], \) left-hand endpoint

\[
\begin{align*}
\pi & \quad \pi & \quad \frac{3\pi}{2} & \quad 2\pi \\
\ldots & \quad \ldots & \quad \ldots & \quad \ldots \\
1 & \quad 2 & \quad 3 & \quad 5
\end{align*}
\]

\( A) \quad B) \quad C) \quad D) \)

**ESSAY.**


Put in \( x^2 \) in "Graph of Riemann sum"; then 0 to 4 in "as \( x \) goes from" 0; Choose Left for "taking samples at", then:

a) Use 4 rectangles, hit "Replot" and write down the resulting picture and state the resulting Estimated area and the Actual area.
b) Use 8 rectangles, hit "Replot" and write down the resulting picture and state the resulting Estimated area and the Actual area.
c) Use 25 rectangles, hit "Replot" then state the resulting Estimated area and the Actual area.
d) Use 100 rectangles, hit "Replot" then state the resulting Estimated area and the Actual area.
e) What is the first number of rectangles that makes the Estimated area be closer than 0.1 away from the Actual area?
21) Go to http://mathworld.wolfram.com/RiemannSum.html
Put in x^2 in "Graph of Riemann sum"; then 0 to 4 in "as x goes from" 0; Choose Right for "taking samples at", then:

a) Use 4 rectangles, hit "Replot" and write down the resulting picture and state the resulting Estimated area and the Actual area.
b) Use 8 rectangles, hit "Replot" and write down the resulting picture and state the resulting Estimated area and the Actual area.
c) Use 25 rectangles, hit "Replot" then state the resulting Estimated area and the Actual area.
d) Use 100 rectangles, hit "Replot" then state the resulting Estimated area and the Actual area.
e) What is the first number of rectangles that makes the Estimated area be closer than 0.1 away from the Actual area?

22) Go to http://mathworld.wolfram.com/RiemannSum.html
Put in 4-x^2 in "Graph of Riemann sum"; then 0 to 4 in "as x goes from" 0; Choose Left for "taking samples at", then:

a) Use 4 rectangles, hit "Replot" and write down the resulting picture and state the resulting Estimated area and the Actual area.
b) Use 8 rectangles, hit "Replot" and write down the resulting picture and state the resulting Estimated area and the Actual area.
c) Use 25 rectangles, hit "Replot" then state the resulting Estimated area and the Actual area.
d) Use 100 rectangles, hit "Replot" then state the resulting Estimated area and the Actual area.
e) What is the first number of rectangles that makes the Estimated area be closer than 0.1 away from the Actual area?

23) Go to http://mathworld.wolfram.com/RiemannSum.html
Put in Your Own function in "Graph of Riemann sum"; then 0 to 4 in "as x goes from" 0; Choose Left for "taking samples at", then:

a) Use 4 rectangles, hit "Replot" and write down the resulting picture and state the resulting Estimated area and the Actual area.
b) Use 8 rectangles, hit "Replot" and write down the resulting picture and state the resulting Estimated area and the Actual area.
c) Use 25 rectangles, hit "Replot" then state the resulting Estimated area and the Actual area.
d) Use 100 rectangles, hit "Replot" then state the resulting Estimated area and the Actual area.
e) What is the first number of rectangles that makes the Estimated area be closer than 0.1 away from the Actual area?
f) Put in Right for "taking samples at" and repeat parts a – e.

24) Watch and take notes on these videos. They will help you in the next group of problems:
25) The figure below shows the velocity of a car for \(0 \leq t \leq 12\) and the rectangles used to estimate the distance traveled.
(a) Do the rectangles represent a left or a right sum?
(b) Do the rectangles lead to an upper or a lower estimate of the distance traveled?
(c) What is the value of \(n\)?
(d) What is the value of \(\Delta t\)?
(e) Give an approximate value for the estimate.

![Velocity Graph]

26) The figure below shows the velocity, \(v\), of an object (in meters/sec). Estimate the total distance the object traveled between \(t = 0\) and \(t = 6\).

\[ v \text{ (m/sec)} \]

![Velocity Graph]
27) The graphs in 9 and 11 below show the velocity, in cm/sec, of a particle moving along a number line. (Positive velocities represent movement to the right; negative velocities to the left.) Compute the change in position between times $t = 0$ and $t = 5$ seconds.

28) When an aircraft attempts to climb as rapidly as possible, its climb rate decreases with altitude. (This occurs because the air is less dense at higher altitudes.) The table shows performance data for a single-engine aircraft.

<table>
<thead>
<tr>
<th>Altitude (1000 ft)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climb rate (ft/min)</td>
<td>925</td>
<td>875</td>
<td>830</td>
<td>780</td>
<td>730</td>
<td>685</td>
</tr>
<tr>
<td>Altitude (1000 ft)</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Climb rate (ft/min)</td>
<td>635</td>
<td>585</td>
<td>535</td>
<td>490</td>
<td>440</td>
<td></td>
</tr>
</tbody>
</table>

Calculate upper and lower estimates for the time required for this aircraft to climb from sea level to 10,000 ft. **Hint:** First change the climb rates to min/ft instead of ft/min. (925 ft/min will be $1/925 \approx .0011$ min/ft). Then graph Altitude (0 to 10) on the horizontal axis, and Climb rate (min/ft) on the vertical axis. Thus, rectangle sums will be in the units (min/ft)·(ft) = minutes, and those are the units you want.
29) A bicyclist starts from home and rides back and forth along a straight east/west highway. Her velocity is given in the figure below (positive velocities indicate travel toward the east, negative toward the west).
(a) On what time intervals is she stopped?
(b) How far from home is she the first time she stops, and in what direction?
(c) At what time does she bike past her house?
(d) If she maintains her velocity at \( t = 11 \), how long will it take her to get back home?

![Velocity Graph](image)

30) A baseball thrown directly upward at 96 ft/sec has velocity \( v(t) = 96 - 32t \) ft/sec at time \( t \) seconds.
(a) Graph the velocity from \( t = 0 \) to \( t = 6 \).
(b) When does the baseball reach the peak of its flight? How high does it go?
(c) How high is the baseball at time \( t = 5 \)?

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the sum. (This is "summation notation" review we need for the next section, The Definite Integral.)

31) \[ \sum_{k=1}^{7} k \]
A) 7 \hspace{1cm} B) 14 \hspace{1cm} C) 56 \hspace{1cm} D) 28

32) \[ \sum_{k=1}^{6} k^2 - 8 \]
A) 28 \hspace{1cm} B) 83 \hspace{1cm} C) 43 \hspace{1cm} D) 91

The Definite Integral

33-R: Read and take notes on Section 5.2
33) **Write these notes on your paper:**

Suppose \( f \) is continuous for \( a \leq t \leq b \). **The definite integral of \( f \) from \( a \) to \( b \),** written \( \int_{a}^{b} f(t) \, dt \), is the limit of the left-hand or right-hand sums with \( n \) subdivisions of \( a \leq t \leq b \) as \( n \) gets arbitrarily large. In other words,

\[
\int_{a}^{b} f(t) \, dt = \lim_{n \to \infty} \left( \text{Left-hand sum} \right) = \lim_{n \to \infty} \left( \sum_{t=0}^{n-1} f(t_i) \Delta t \right)
\]

and

\[
\int_{a}^{b} f(t) \, dt = \lim_{n \to \infty} \left( \text{Right-hand sum} \right) = \lim_{n \to \infty} \left( \sum_{t=1}^{n} f(t_i) \Delta t \right)
\]

Each of these sums is called a **Riemann sum**, \( f \) is called the **integrand**, and \( a \) and \( b \) are called the **limits of integration**. The “\( \int \)” notation comes from an old-fashioned “\( S \)” which stands for “sum” in the same way that \( \sum \) does. The “\( dt \)” in the integral comes from the factor \( \Delta t \).

34) **Write these notes on your paper:**

**The Definite Integral as an Area:**

a) When \( f(x) \geq 0 \) and \( a < b \):

The area under the graph of \( f \) and above x-axis between \( a \) and \( b \) is

\[ \int_{a}^{b} f(x) \, dx \]

b) When \( f(x) \) Is Not Positive: When \( f(x) \) is positive for some \( x \) values and negative for others, and \( a < b \):

\[ \int_{a}^{b} f(x) \, dx \] is the sum of areas above the x-axis, counted positively, and areas below the x-axis, counted negatively.

35) **Write these notes on your paper:**

FYI: The **general Riemann sum** for \( f \) on the interval \([a,b]\) is a sum of the form

\[ \sum_{i=1}^{n} f(c_i) \Delta t_i \]

where \( a = t_0 < t_1 < \cdots < t_n = b \), and, for \( i = 1, \ldots, n \), \( \Delta t_i = (t_i) - (t_{i-1}) \), and \( t_{i-1} \leq c_i \leq t_i \). In other words, instead of \( \Delta t \) being constant, the \( \Delta t \) intervals are of varying lengths and \( f \) is evaluated at some point in the interval, not just at the endpoints. Note: It is best to first learn Riemann sums as left and right sums (as we have done in this class) before graduating up to the general form which is for senior level Advanced Calculus.

36) **Watch and take notes on these videos**  
   [http://youtu.be/Cfo0n8LyqN8](http://youtu.be/Cfo0n8LyqN8),  
37) **Load in** the "Integral" program from my calculator to yours (done in class) — You must have this program or a similar version of it in your TI-84 because you will be at a disadvantage on tests if you do not have it.

38) *(optional)* There is also a great **Riemann sum program** for the TI-84. Go to http://www.calcblog.com/riemann-sum-program-ti83-ti84/ There you see a nice tutorial about Riemann sums then you scroll down to the Using the Riemann Program section. It shows how to download the program into your TI-84 from your computer, and various links show you how to get the program into your computer in the first place. (I did it and it worked!) Then it shows how to use the program.


40) The figure below shows a Riemann sum approximation with n subdivisions to \( \int_{a}^{b} f(x) \, dx \)

(a) Is it a left- or right-hand approximation? Would the other one be larger or smaller?

(b) What are a, b, n and \( \Delta x \)?
41) **Do 5, 7, 9.** Use the **Integral program** in your calculator to find the value of the definite integral (use \( n = 100, \text{TRAP} \)) Note: The Trapezoid method is the average of the Left and Right sums.

(These problems are from Section 5.2)

\[
5. \quad \int_{1}^{4} (x^2 + x) \, dx
\]

\[
7. \quad \int_{-1}^{1} e^{-x^2} \, dx
\]

\[
9. \quad \int_{0}^{1} \sin(t^2) \, dt
\]

42) Use the figure to estimate \( \int_{0}^{20} f(x) \, dx \)

43) Using the figure, estimate \( \int_{-3}^{5} f(x) \, dx \)
44) (a) What is the area between the graph of \( f(x) \) in the figure and the x-axis, between \( x = 0 \) and \( x = 5 \)?
(b) What is \( \int_{0}^{5} f(x) \, dx \)?

45) (a) Find the total area between \( f(x) = x^3 - x \) and the x-axis for \( 0 \leq x \leq 3 \).
(b) Find \( \int_{0}^{3} f(x) \, dx \)
(c) Are the answers to parts (a) and (b) the same? Explain.
46) **Do 23, 25, 27, 29, 31. Find the area under the given curve** (For 23,25,27 use your Integral program (n=100, TRAP). (These problems are from Section 5.2)

23. Under the curve \( y = \cos t \) for \( 0 \leq t \leq \pi/2 \).
24. Under \( y = \ln x \) for \( 1 \leq x \leq 4 \).
25. Under \( y = 2 \cos(t/10) \) for \( 1 \leq t \leq 2 \).
26. Under the curve \( y = \cos \sqrt{x} \) for \( 0 \leq x \leq 2 \).
27. Under the curve \( y = 7 - x^2 \) and above the \( x \)-axis.
28. Above the curve \( y = x^4 - 8 \) and below the \( x \)-axis.
29. Use Figure 5.36 to find the values of
   
   \[
   \begin{align*}
   (a) & \quad \int_a^b f(x) \, dx \\
   (b) & \quad \int_b^c f(x) \, dx \\
   (c) & \quad \int_a^c f(x) \, dx \\
   (d) & \quad \int_a^c |f(x)| \, dx
   \end{align*}
   \]

![Figure 5.36](image)

**Figure 5.36**

31. (a) Using Figure 5.38, find \( \int_{-3}^{0} f(x) \, dx \).
   
   (b) If the area of the shaded region is \( A \), estimate \( \int_{-3}^{4} f(x) \, dx \).

![Figure 5.38](image)
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Graph the integrand and use areas to evaluate the integral. Then run the Integral program with \( n = 100 \), sub intervals (and TRAP) to verify your answer. (\( n = 100 \) is a large enough amount of intervals to give a pretty close estimate for these problems below because the widths of the intervals are not too large. (Only the problem with \( \sqrt{16 - x^2} \) may have an answer that is significantly far off from the Riemann estimate with \( n=100 \) sub intervals – learn why in class)

47) \( \int_{-1}^{7} 4 \, dx \)
   A) 32       B) 8       C) 24       D) 16

48) \( \int_{6}^{10} x \, dx \)
   A) 8       B) 16       C) 64       D) 32

49) \( \int_{0}^{4} 8x \, dx \)
   A) 128       B) 8       C) 32       D) 64

50) \( \int_{-5}^{5} (2x + 10) \, dx \)
   A) 100       B) 20       C) 50       D) 200

51) \( \int_{-2}^{9} |x| \, dx \)
   A) \( \frac{85}{2} \)       B) 85       C) 11       D) \( \frac{77}{2} \)

52) \( \int_{-4}^{4} \sqrt{16 - x^2} \, dx \) (Graph it - It’s a semicircle so you know the area by geometry)
   A) \( 4\pi \)       B) \( 16\pi \)       C) 16       D) \( 8\pi \)

**The Fundamental Theorem of Calculus**

53–R: Read and take notes on Section 5.3
ESSAY.

53) Write these notes on your paper:
The Fundamental Theorem of Calculus:

If \( f \) is continuous on the interval \([a,b]\) and \( f(t) = F'(t) \), then
\[
\int_a^b f(t) \, dt = F(b) - F(a).
\]

The F.T.O.C says that if you can find an antiderivative \( F(t) \) for your function \( f(t) \), then instead of computing area estimates or limits of sums to find \( \int_a^b f(t) \, dt \), you can just subtract \( F(b) - F(a) \). A very simple subtraction! And this theorem ties the derivative (\( F'(t) = f(t) \)) to sums (\( \sum f(t) \Delta t \)) in a very slick way! Think of the F.T.O.C. this way: In the distance–rate–time model we started off integration with, we took the product of velocity values and small time increments and that gave us small changes in position that we then added up to get the total change in position. Well, the total change in position over the interval is just the position at b minus the position at a, that is, \( F(b) - F(a) \). So it really, it is a simple outcome! Profound and simple, those are the things that last through time!

54) Know the proof steps of The Fundamental Theorem of Calculus as shown in the following video for a test – 100% guaranteed to be on the test. The proof steps are explained and organized into 8 steps:

http://youtu.be/brSw4kgogJw

55) a) Find \( \int_1^3 2x \, dx \) by geometry, and then by using the Integral program with \( n = 100 \), TRAP

b) Guess at an antiderivative, \( F(x) \), for \( f(x) = 2x \) such that \( F'(x) = 2x \)

c) Using your antiderivative function \( F(x) \) you found in (b), compute \( F(3) - F(1) \)

d) Does your answer answer to (c) match with your answer to (a)? If so, you have just supported the FTOC!

56) The rate at which the world’s oil is being consumed is \( r(t) = 32e^{0.05t} \) billion barrels per year where \( t = \) years after 2004.  http://youtu.be/MVhjfYMw-A

a) Set up a definite integral that will give the total amount of barrels of oil consumed from 2004 to 2012 (assuming the rate functions remains accurate over that time)

b) Find(estimate) the value of the integral by using the integral program (\( n = 100 \), TRAP)

c) Now you want to answer the problem by using the Fundamental Theorem of Calculus but you need an antiderivative (a function whose derivative is \( 32e^{0.05t} \)), call it \( F(t) \), so you can find \( F(b) - F(a) \). Go to http://www.wolframalpha.com and put in: "antiderivative 32e^{0.05t} " and an antiderivative shows up.

20
57) A spill of radioactive iodine occurs and the rate of decay is given by \( r(t) = 2.4e^{-0.004t} \) millirems/hour where \( t = 0 \) hours is the time of the spill.
   a) An acceptable rate of radiation is 0.6 millirems/hour. How many hours will it take to reach that rate?
   b) Set up a definite integral that will give the total amount of radiation (in millirems) that will be emitted from \( t = 0 \) hours to the time of the acceptable rate level?
   c) Find (estimate) the value of the integral by using technology methods with \( n = 500 \) intervals
   c) Now you want to answer the problem by using the Fundamental Theorem of Calculus but you need an antiderivative (a function whose derivative is \( 2.4e^{-0.004t} \)), call it \( F(t) \), so you can find \( F(b) - F(a) \). Go to http://www.wolframalpha.com and put in: "antiderivative 2.4e^( -0.004t) " and an antiderivative shows up.

58) A runner is jogging on a straight track and is speeding up and slowing down with velocity function \( v(t) = \cos(t) + 5 \) miles/hr where \( t \) is the hours after he/she started.
   a) Set up a definite integral that will give the total distance he/she ran from the time period of \( t = 1 \) hour to \( t = 2 \) hours.
   b) Find (estimate) the value of the integral by using technology methods with \( n = 100 \) intervals
   c) Now you want to answer the problem by using the Fundamental Theorem of Calculus but you need an antiderivative (a function whose derivative is \( \cos(t)+5 \)), call it \( F(t) \), so you can find \( F(b) - F(a) \). Go to http://www.wolframalpha.com and put in: "antiderivative \( \cos(t)+5 \) " and an antiderivative shows up.

59) Water is leaking out of a tank at a rate of \( R(t) \) gallons/hour, where \( t \) is measured in hours.
   (a) Write a definite integral that expresses the total amount of water that leaks out in the first two hours.
   (b) In the figure below, shade the region whose area represents the total amount of water that leaks out in the first two hours.
   (c) Using the figure, give an estimate of the total amount of water that leaks out in the first two hours.

![Graph of R(t) vs t]

60) The rate at which the world’s oil is consumed (in billions of barrels per year) is given by \( r = f(t) \), where \( t \) is in years and \( t = 0 \) is the start of 2004.
   (a) Write a definite integral representing the total quantity of oil consumed between the start of 2004 and the start of 2009.
   (b) Between 2004 and 2009, the rate was modeled by \( r = 32e^{0.05t} \). Using a left-hand sum with five subdivisions, find an approximate value for the total quantity of oil consumed between the start of 2004 and the start of 2009.
61) The amount of waste a company produces, \( W \), in tons per week, is approximated by \( W = 3.75e^{-0.008t} \), where \( t \) is in weeks since January 1, 2005. Waste removal for the company costs $15/ton. How much does the company pay for waste removal during the year 2005?

62) The formula for the **standard normal distribution** from statistics, is given by

\[
f(z) = \frac{-z^2}{2\sqrt{2\pi}} e^{\frac{-z^2}{2}}.
\]

This is the famous **bell shaped curve**. Graph it on \( z: -5 \) to \( 5 \), \( y: 0 \) to \( 0.5 \) and see. One context in which it is used is for the distribution of IQ scores. In this context IQ scores are normalized such that an IQ of 100 corresponds to a \( z \) score of 0. In the IQ context the area under this curve from \( z = a \) to \( z = b \) gives the proportion of people with a normalized IQ score between \( z = a \) and \( z = b \).

a) A person is said to have "average" IQ if the \( z \) score for his/her IQ is between \(-1\) and \(1\). What is the percentage of people with "average" IQ? (convert the decimal to percent)
b) A person is said to be "gifted" if the \( z \) score for his/her IQ is between 2 and 3. What is the percentage of people who are "gifted"? (convert the decimal to percent)
c) A person is said to be a genius if the \( z \) score for his/her IQ is above 3 (from 3 to \(\infty\)). Let’s assume that 10 is far enough out. What is the percentage of people who are geniuses (that have normalized IQ from 3 to 10)? (convert the decimal to percent)

63) A cup of coffee at 90 degrees C is put into a 20 degrees C room when \( t = 0 \). The coffee’s temperature is changing at a rate of \( r(t) = -7e^{-0.1t} \) degrees C per minute, with \( t \) in minutes. Estimate the coffee’s temperature when \( t = 10 \).

### Average Value of a function on an interval

64) Go to [http://calculusapplets.com/aveval.html](http://calculusapplets.com/aveval.html) and click "hide answer". Put in \( f(x) = x^2 \) and put in \( ymax = 5 \) and hit enter. You want to find the average value of \( f(x) \) on \( a = 0 \) to \( b = 2 \). Think of the function as a velocity of a man walking in ft/sec. Think of the average value of the function as the constant velocity he could have walked to cover the same ground as if he walked according to the function.

a) Click on the black square at the origin and move it up to what you think the average value would be. (Another way to think of it is: Move the black square up to a point where the area under the resulting rectangle equals the yellow area under the \( f(x) \) curve.) What is your average value guess?
b) Click "show answer" and see what the applet says. Write that down.
c) Compute the average value of \( f(x) = x^2 \) on \( x = 0 \) to \( x = 2 \) by using the Average Value of \( f \) formula:

\[
\text{Average value of } f \text{ on the interval } a \text{ to } b = \frac{1}{b-a} \int_{a}^{b} f(x) \, dx.
\]

Use \( F(x) = \frac{x^3}{3} \) as an antiderivative of \( f(x) = x^2 \). Did the answer match with the applet’s answer for the average value?

65) [Watch and take notes on this video](http://youtu.be/XyN34iSqCRI) showing the derivation of the formula for the average value of a function on an interval. Know the derivation of the average value formula for a test!
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the average value of the function over the given interval. If you can think of an antiderivative for the function then use it, if not, go to http://www.wolframalpha.com and put in: “antiderivative {the function}”

66) \( f(x) = 4x \) on \([5, 7]\)
   A) 12  
   B) 48  
   C) 24  
   D) 96

67) \( y = 6 - x^2 \); \([-5, 2]\)  
   A) \(-\frac{1}{3}\)  
   B) \(-\frac{13}{3}\)  
   C) 3  
   D) \(\frac{81}{7}\)

68) \( y = x^2 - 2x + 4 \); \([0, 2]\)
   A) 3  
   B) \(\frac{22}{3}\)  
   C) 4  
   D) \(\frac{10}{3}\)

ESSAY.

69) The value, \( V \), of a Tiffany lamp, worth $225 in 1975, increases at 15% per year. Its value in dollars \( t \) years after 1975 is given by \( V = 225(1.15)^t \). Find the average value of the lamp over the period 1975–2010.

70) (a) Using the figure below, find \( \int_1^6 f(x) \, dx \)

(b) What is the average value of \( f \) on \([1,6]\)?

![Graph of f(x) with x values: 1, 2, 3, 4, 5, 6]

**Integral properties**

71-R: Read and take notes on Section 5.4

71) Write these notes on your paper:
   If \( a, b, \) and \( c \) are any numbers and \( f \) is a continuous function, then

1) \( \int_a^b f(x) \, dx = -\int_b^a f(x) \, dx \). This means: The integral from \( b \) to \( a \) is the negative of the integral from \( a \) to \( b \). Think of a Riemann sum and consider the x values going from \( b \) backward to \( a \). That would mean negative \( \Delta x \) increments are being used: \( \Delta x = (a - b)/n = -(b - a)/n \). So when all the \( f(x) \Delta x \) quantities are added up, the result is the negative of the sum produced by going forward from \( a \) to \( b \).
2) \[ \int_a^c f(x) \, dx + \int_c^b f(x) \, dx = \int_a^b f(x) \, dx \]. This means: The integral from a to c plus the integral from c to b is the integral from a to b. Just think of sums. You can break up a sum into parts and the result stays equal. The illustration below shows that the area under the curve from a to c added to the area from c to b equals the area from a to b.

![Graph](image)

3) \[ \int_a^b (f(x) \pm g(x)) \, dx = \int_a^b f(x) \, dx \pm \int_a^b g(x) \, dx \]. This means: The integral of the sum (or difference) of two functions is the sum (or difference) of their integrals. Integrals are sums and we can sum up things in different orders. So when considering \((f(x) + g(x)) \Delta x\), we can sum up all the \(f(x) \Delta x\) values together and then add on the sum of all the \(g(x) \Delta x\) values and get the same result. Or you can think of it as the rectangle areas from \(g(x)\) are stacked on top of the rectangle areas from \(f(x)\) so that the area under \(f(x) + g(x)\) is the same as the area under \(f(x)\) plus the area under \(g(x)\) as this picture shows:

![Graph](image)

4) \[ \int_a^b cf(x) \, dx = c \int_a^b f(x) \, dx \]. This means: The integral of a constant times a function is that constant times the integral of the function. Integrals are sums and we can factor out a constant multiplier from a sum.
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

72) Suppose that \( \int_{1}^{2} f(x) \, dx = -2 \). Find \( \int_{1}^{2} 6f(u) \, du \) and \( \int_{1}^{2} -f(u) \, du \).  

A) 4; -2  
B) 6; 2  
C) -12; -\frac{1}{2}  
D) -12; 2

73) Suppose that \( \int_{-4}^{-1} g(t) \, dt = -12 \). Find \( \int_{-4}^{-1} \frac{g(x)}{-12} \, dx \) and \( \int_{-1}^{-4} -g(t) \, dt \).

A) 1; 12  
B) 1; -12  
C) 0; -12  
D) -1; 12

74) Suppose that \( f \) and \( g \) are continuous and that \( \int_{4}^{8} f(x) \, dx = -5 \) and \( \int_{4}^{8} g(x) \, dx = 9 \).

Find \( \int_{4}^{8} [4f(x) + g(x)] \, dx \).

A) 13  
B) -11  
C) 16  
D) 31

75) Suppose that \( f \) and \( g \) are continuous and that \( \int_{2}^{6} f(x) \, dx = -2 \) and \( \int_{2}^{6} g(x) \, dx = 7 \).

Find \( \int_{2}^{6} [f(x) - 2g(x)] \, dx \).

A) -18  
B) 12  
C) -16  
D) -9

76) Suppose that \( f \) and \( g \) are continuous and that \( \int_{2}^{6} f(x) \, dx = -5 \) and \( \int_{2}^{6} g(x) \, dx = 7 \).

Find \( \int_{2}^{6} [g(x) - f(x)] \, dx \).

A) 12  
B) -2  
C) -12  
D) 2

77) Suppose that \( h \) is continuous and that \( \int_{-4}^{5} h(x) \, dx = 7 \) and \( \int_{-4}^{6} h(x) \, dx = -9 \). Find \( \int_{-4}^{6} h(t) \, dt \) and \( \int_{-4}^{6} h(t) \, dt \).

A) -2; 2  
B) 2; -2  
C) 16; -16  
D) -16; 16
78) Suppose that \( f \) is continuous and that \( \int_{-4}^{4} f(z) \, dz = 0 \) and \( \int_{-4}^{4} f(z) \, dz = 4 \). Find \( \int_{4}^{-4} 2f(x) \, dx \).

A) \(-2\)  
B) \(8\)  
C) \(-8\)  
D) \(-4\)

Evaluate the integral. Think of antiderivative or use Wolframalpha to get one, then use the FTOC. (You can use the Integral program to verify your answer but you must use the FTOC first.)

79) \( \int_{1}^{\sqrt[13]{13}} x \, dx \)  

A) \(\sqrt[13]{13} - 1\)  
B) \(12\)  
C) \(-6\)  
D) \(6\)

80) \( \int_{\frac{3\pi}{2}}^{2\pi} \theta \, d\theta \)  

A) \(\frac{9\pi^2}{8}\)  
B) \(\frac{7\pi^2}{8}\)  
C) \(\frac{\pi^2}{2}\)  
D) \(\frac{\pi^2}{8}\)

81) \( \int_{0}^{\frac{1}{8}} t^2 \, dt \)  

A) \(\frac{1}{1536}\)  
B) \(-\frac{1}{1536}\)  
C) \(-\frac{1}{8}\)  
D) \(1536\)

82) \( \int_{0}^{\sqrt[13]{13}} x^2 \, dx \)  

A) \(169\)  
B) \(\frac{3\sqrt{13}}{3}\)  
C) \(\frac{13\sqrt{13}}{3}\)  
D) \(\frac{13}{3}\)

83) \( \int_{\pi}^{2\pi} \theta^2 \, d\theta \)  

A) \(\frac{\pi^3}{3}\)  
B) \(\frac{27\pi^3}{24}\)  
C) \(\frac{7\pi^3}{3}\)  
D) \(\frac{\pi^3}{24}\)

84) \( \int_{2}^{\sqrt{15}} (z - \sqrt{15}) \, dz \)  

A) \(-\frac{19}{2}\sqrt{15}\)  
B) \(-\frac{19}{2} + 2\sqrt{15}\)  
C) \(-\frac{15}{2}\sqrt{15}\)  
D) \(-\sqrt{15}\)
85) \( \int_{8}^{12} \frac{7}{x} \, dx \)

A) 84  
B) 28  
C) -76  
D) 0

**Finding graphs and values of antiderivatives by the Slope Field method and the FTOC method**

**86–R:** Read and take notes on Section 6.1

**ESSAY.**

86) **Watch and take notes on these videos:**  
   - Slope Field method: [http://youtu.be/QH1uGXjWB5U](http://youtu.be/QH1uGXjWB5U)  
   - [http://youtu.be/Pz52Kldnl6s](http://youtu.be/Pz52Kldnl6s)  
   - [http://youtu.be/Ap5NqBOgD7s](http://youtu.be/Ap5NqBOgD7s)  
   These will help you for the next section of problems.
87) **Do 5, 7, 9, 11** Sketch two functions $F$ such that $F' = f$. In one case let $F(0) = 0$ and in the other, let $F(0) = 1$. Use the slope field method AND the FTOC method on each. *(These problems are from Section 6.1)*

5. 

$$
\begin{align*}
\text{x} & \\
\text{f(x)} & \\
-1 & \\
1 & \\
1 & \\
\end{align*}
$$

7. 

$$
\begin{align*}
\text{x} & \\
\text{f(x)} & \\
-1 & \\
1 & \\
\end{align*}
$$

9. 

$$
\begin{align*}
\text{x} & \\
\text{f(x)} & \\
-1 & \\
1 & \\
\end{align*}
$$

11. 

$$
\begin{align*}
\text{x} & \\
\text{f(x)} & \\
-1 & \\
1 & \\
\end{align*}
$$

88) Estimate $f(x)$ for $x = 2, 4, 6$, using the given values of $f'(x)$ and the fact that $f(0) = 100$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'(x)$</td>
<td>10</td>
<td>18</td>
<td>23</td>
<td>25</td>
</tr>
</tbody>
</table>
89) The graph of $\frac{dy}{dt}$ against $t$ is in the figure below. The three shaded regions each have area 2. If $y = 0$ when $t = 0$, draw the graph of $y$ as a function of $t$, labeling the known $y$-values, maxima and minima, and inflection points. Mark $t_1, t_2, ..., t_5$ on the $t$ axis.

90) Using the figure below, sketch a graph of an antiderivative $G(t)$ of $g(t)$ satisfying $G(0) = 5$. Label each critical point of $G(t)$ with its coordinates.
91) The graph in the figure records the spillage rate at a toxic waste treatment plant over the 50 minutes it took to plug the leak.

(a) Complete the table for the total quantity spilled in liters in time t minutes since the spill started.

<table>
<thead>
<tr>
<th>Time t (min)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity (liters)</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Graph the total quantity leaked against time for the entire fifty minutes. Label axes and include units.

**Finding Antiderivatives by hand (analytically, or, algebraically)**

92-R: Read and take notes on Section 6.2
Write these notes on your paper:
1) If \( F'(x) = 0 \) on an interval, then \( F(x) = C \) on this interval, for some constant \( C \).

2) If \( F \) and \( G \) are both antiderivatives of \( f \) on an interval, then \( G(x) = F(x) + C \). This means that all antiderivatives for a particular \( f(x) \) function are an added constant from each other. So if \( F(x) \) is an antiderivative for \( f(x) \) then ALL antiderivatives of \( f(x) \) are of the form \( F(x) + C \).

3) \( \int f(x) \, dx \) is called the **indefinite integral** of \( f(x) \) and is the notation we use for an antiderivative of \( f(x) \). So when a problem says to find \( \int f(x) \, dx \), it means to find an antiderivative for \( f(x) \).

4) If \( k \) is constant, \( \int k \, dx = kx + C \) (To believe this just take the derivative of \( kx + C \) and you see you get \( k \). So that is the proof of the statement!)

5) \( \int x^n \, dx = \frac{x^{n+1}}{n+1} + C \), \( n \neq -1 \). To believe this, just take the derivative of \( \frac{x^{n+1}}{n+1} + C \) and see that you get \( x^n \). **Note, if \( n = -1 \) then we have trouble** with this formula, that is, the fraction is undefined. If \( n = -1 \) we see that the function is \( x^{-1} \) which is \( \frac{1}{x} \). The next formula addresses this.

6) \( \int \frac{1}{x} \, dx = \ln|x| + C \). We already know that \( (\ln(x))' = \frac{1}{x} \) but **where does the absolute value in \( \ln|x| \) come from?** Watch this animation video to see why: [http://youtu.be/p-Kr9xLBzMc](http://youtu.be/p-Kr9xLBzMc)

7) \( \int e^x \, dx = e^x + C \), and \( \int e^{ax} \, dx = \frac{1}{a} e^{ax} + C \). Take the derivatives of these antiderivatives and you see you get back to the integrand functions.

8) \( \int \cos(x) \, dx = \sin(x) + C \) and \( \int \sin(x) \, dx = -\cos(x) + C \)

Write these notes on your paper:
**Sums and Constant Multiples for indefinite integrals:**

1) \( \int (f(x) \pm g(x)) \, dx = \int f(x) \, dx + \int g(x) \, dx \). That is, an antiderivative of the sum (or difference) of two functions is the sum (or difference) of their antiderivatives. This follows directly from the sum rules for derivatives: \( (\int f(x) \, dx + \int g(x) \, dx)' = (\int f(x) \, dx)' + (\int g(x) \, dx)' = f(x) + g(x) \) therefore \( \int f(x) \, dx + \int g(x) \, dx \) is an antiderivative for \( f(x) + g(x) \), thus \( \int f(x) \, dx + \int g(x) \, dx = \int (f(x) + g(x)) \, dx \)

2) \( \int c f(x) \, dx = c \int f(x) \, dx \). That is, an antiderivative of a constant times a function is the constant times an antiderivative of the function. This follows directly from the constant multiple rule for derivatives.

95) Find antiderivatives for 1–15 odd: (These problems are from Section 6.2)

1. \( f(x) = 5 \)
3. \( f(x) = x^2 \)
5. \( g(z) = \sqrt{z} \)
7. \( r(t) = \frac{1}{t^2} \)
9. \( g(z) = \frac{1}{z^3} \)
11. \( f(z) = e^z \)
13. \( f(t) = 2t^2 + 3t^3 + 4t^4 \)
15. \( f(t) = \frac{t^2 + 1}{t} \)

96) Find the general antiderivative for 17 – 27 odd (get an antiderivative then just add on the constant C): (These problems are from Section 6.2)

17. \( f(t) = 6t \)
19. \( f(x) = x^2 - 4x + 7 \)
21. \( r(t) = t^3 + 5t - 1 \)
23. \( g(x) = \sin x + \cos x \)
25. \( p(t) = \frac{1}{\sqrt{t}} \)
27. \( g(x) = \frac{5}{x^3} \)
97) Find the indefinite integrals for 37–49 odd: (These problems are from Section 6.2)

37. \( \int (5x + 7) \, dx \)

39. \( \int (2 + \cos t) \, dt \)

41. \( \int (3e^x + 2\sin x) \, dx \)

43. \( \int (5x^2 + 2\sqrt{x}) \, dx \)

45. \( \int \frac{8}{\sqrt{x}} \, dx \)

47. \( \int (e^x + 5) \, dx \)

49. \( \int \left( \sqrt{x^3} - \frac{2}{x} \right) \, dx \)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral. By hand, then check by running the Integral program.

98) \( \int_{-2}^{6} 6x^5 \, dx \)
   A) 1280  B) 279,552  C) -46,592  D) 46,592

99) \( \int_{0}^{16} 2\sqrt{x} \, dx \) \( \text{http://youtu.be/dp_0v3cP9ec} \)
   A) 128  B) 192  C) 16  D) \( \frac{256}{3} \)
100) \( \int_{1}^{4} \frac{t^2 + 1}{\sqrt{t}} \, dt \) \hspace{1cm} \text{(Hint: first simplify the integrand by dividing the } \sqrt{t} \text{ into both terms of } t^2 + 1 \)

A) \( \frac{92}{5} \) \hspace{1cm} B) \( \frac{77}{5} \) \hspace{1cm} C) \( \frac{72}{5} \) \hspace{1cm} D) 32

101) \( \int_{0}^{\pi/2} 18 \sin x \, dx \)

A) 0 \hspace{1cm} B) 1 \hspace{1cm} C) 18 \hspace{1cm} D) -18

ESSAY.

102) Find the definite integrals for 51 – 59 odd by hand (check by running the Integral program): (These problems are from Section 6.2)

51. \( \int_{0}^{3} (x^2 + 4x + 3) \, dx \)

53. \( \int_{0}^{\pi/4} \sin x \, dx \)

55. \( \int_{0}^{2} 3e^x \, dx \)

57. \( \int_{0}^{1} \sin \theta \, d\theta \)

59. \( \int_{0}^{2} \left( \frac{x^3}{3} + 2x \right) \, dx \)

103) Water is pumped into a cylindrical tank, standing vertically, at a decreasing rate given at time t minutes by \( r(t) = 120 - 6t \) ft³/min for \( 0 \leq t \leq 10 \). The tank has radius 5 ft and is empty when \( t = 0 \). Find the depth of water in the tank at \( t = 4 \).

104) Use the Fundamental Theorem (FTOC) to find the area under \( f(x) = x^2 \) between \( x = 0 \) and \( x = 3 \). That is, set up the definite integral and evaluate by hand, then check with the Integral program.
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the total area of the region between the curve and the x-axis. By hand, then check with the Integral program.

105) \( y = 2x + 7; \ 1 \leq x \leq 5 \)
   A) 52   B) 18   C) 26   D) 9

106) \( y = 2x - x^2; \ 0 \leq x \leq 2 \)
   A) \( \frac{2}{3} \)   B) \( \frac{4}{3} \)   C) \( \frac{7}{3} \)   D) \( \frac{5}{3} \)

107) \( y = \frac{3}{x^3}; \ 1 \leq x \leq 3 \)
   A) \( \frac{1}{3} \)   B) \( \frac{4}{3} \)   C) \( \frac{1}{2} \)   D) 3

108) \( y = -x^2 + 9; \ 0 \leq x \leq 5 \) [http://youtu.be/tuYZvBGshis]
   A) \( \frac{98}{3} \)   B) \( \frac{5}{9} \)   C) \( \frac{10}{9} \)   D) \( \frac{10}{3} \)

109) \( y = \frac{1}{\sqrt{x}}; \ 1 \leq x \leq 4 \)
   A) 2   B) 4   C) \( \frac{1}{4} \)   D) \( \frac{1}{2} \)

Find the average value of the function over the given interval. (By hand, then check with the Integral program.)

110) \( y = x^2 - 2x + 6; \ [0, 2] \) [http://youtu.be/yIxMqC5iL08]
   A) \( \frac{16}{3} \)   B) 5   C) 6   D) \( \frac{34}{3} \)

111) \( y = 3 - x^2; \ [-2, 2] \)
   A) 3   B) \( \frac{5}{3} \)   C) \( -\frac{1}{3} \)   D) 0

**Area enclosed by curves**

ESSAY.

112) Watch and take notes on this video about finding the area between curves. This will help you for the next group of problems: [http://youtu.be/3LV4i2gQaOM]
Find the area of the regions in 13 – 19 odd. Show a graph for each. (These problems are from Section 5.4)

13. Under \( y = e^x \) and above \( y = 1 \) for \( 0 \leq x \leq 2 \).
14. Under \( y = 5 \ln(2x) \) and above \( y = 3 \) for \( 3 \leq x \leq 5 \).
15. Between \( y = x^2 \) and \( y = x^3 \) for \( 0 \leq x \leq 1 \).
16. Between \( y = x^{1/2} \) and \( y = x^{1/3} \) for \( 0 \leq x \leq 1 \).
17. Between \( y = \sin x + 2 \) and \( y = 0.5 \) for \( 6 \leq x \leq 10 \).
18. Between \( y = \cos t \) and \( y = \sin t \) for \( 0 \leq t \leq \pi \).
19. Between \( y = e^{-x} \) and \( y = 4(x - x^2) \).

**Construction of Antiderivatives** for functions we can’t use any techniques to obtain. (There are still more techniques of integration to learn but still there will be functions for which we just cannot obtain a closed form antiderivative – we use the construction method to find values of the antiderivative function in that case.)

114–R: Read and take notes on Section 6.4

114) Write these notes on your paper:

Construction Theorem for Antiderivatives (**Second Fundamental Theorem of Calculus**):

If \( f \) is a continuous function on an interval, and if \( a \) is any number in that interval, then the function \( F \) defined on the interval as follows is an antiderivative of \( f \):

\[
F(x) = \int_{a}^{x} f(t) \, dt
\]

This means: Whenever we are stuck for an antiderivative of a function \( f(x) \), we can always construct one (get all the values we want for one) by using \( \int_{a}^{x} f(t) \, dt \). You simply pick a value of \( a \) where you want the antiderivative to be 0, then plug in \( x \) values and evaluate the integral for each to get the antiderivative values.

115) Watch and take notes on this video for an explanation and proof of the 2nd FTOC:

[http://youtu.be/R7E1RfDScjo](http://youtu.be/R7E1RfDScjo)

116) a) Evaluate \( \int_{a}^{x} 2t \, dt \) using the FTOC and call the result \( F(x) \). ("a" is any constant)

b) Take the derivative of \( F(x) \) you found in part (a) and show you get \( 2x \) and thus demonstrating that \( \int_{a}^{x} 2t \, dt \) is an antiderivative for \( 2x \).
117) a) Make a table of values of \( F(x) = \int_{0}^{x} 2t \, dt \) for \( x = 0, 1, 2, 3, 4 \) by using your Integral program with \( n=100 \) sub-intervals to evaluate the integral at each \( x \) value, one at a time.

b) Graph your table of values. Does the graph of \( F(x) \) look like it’s derivative function would be \( 2x \) ?

118) a) Make a table of values of \( F(x) = \int_{0}^{x} e^{-t^2} \, dt \) for \( x = 0, .2, .4, .6, .8, 1 \) by using your Integral program with \( n=100 \) sub-intervals (use TRAP) to evaluate the integral at each \( x \) value.

b) Graph your table of values. Does the graph of \( F(x) \) look like it’s derivative function would be \( e^{-x^2} \) ?

http://youtu.be/7-2FZ4c89fc

119) Watch and take notes on this video. It will help you for the next group of problems.

http://youtu.be/AGqzDV8IRm8

120) Write an expression for the function, \( f(x) \), with the given property:

\[ f'(x) = \frac{\sin x}{x} \text{ and } f(1) = 5 \]

121) Find the derivatives for 11, 13, 15: (These problems are from Section 6.4)

11. \[ \frac{d}{dx} \int_{0}^{x} \cos(t^2) \, dt \]

13. \[ \frac{d}{dx} \int_{1}^{x} (1 + t)^{200} \, dt \]

15. \[ \frac{d}{dx} \int_{0.5}^{x} \arctan(t^2) \, dt \]
122) Use the **chain rule** to calculate the derivative for 35, 37 (for 37, first reverse the limits of integration and account for that by putting a negative sign outside of the integral). (These problems are from Section 6.4)

\[
35. \quad \frac{d}{dx} \int_{0}^{x^2} \ln(1 + t^2) \, dt
\]

\[
37. \quad \frac{d}{dt} \int_{2t}^{4} \sin(\sqrt{x}) \, dx
\]

**Equations of motion**

123) **Watch and take notes on this video.** It shows the derivation steps of how we start with the acceleration of an object due to gravity, \(-g\), and end up knowing the position function of the object, \(s(t)\). Know for a test. [http://youtu.be/p6IIZ9KO3iQ](http://youtu.be/p6IIZ9KO3iQ)

124) On the moon the acceleration due to gravity is 5 ft/sec\(^2\). An astronaut jumps upward from the ground with initial velocity 10 ft/sec.
   a) Express his height, \(h(t)\), in feet above the ground at time, \(t\), in seconds.
   b) How high does he go?
   c) How long is the astronaut off the ground?

125) At time \(t = 0\) sec. a stone is thrown straight down off a 250 ft. cliff with initial velocity 20 ft/sec.
   Acceleration due to gravity is 32 ft/sec\(^2\). Express its height, \(h(t)\), in feet above the ground at time, \(t\), in seconds. Note: since \(h(t)\) is from the ground up, and the the stone was thrown downward, opposite of this direction it means the initial velocity is actually \(-20\) ft/sec for the purpose of building up the \(h(t)\) function. And of course gravity is opposite of the direction of the height so it also is negative. Refer to the video in the previous problem for help.

**Do these next 3 problems about John Carter – good test question!**

126) **Watch this video.** In the movie, "**John Carter** of Mars", a civil war era person is transported to Mars. The first thing he encounters is that gravity is far less on Mars than on Earth. [http://www.youtube.com/watch?v=C5LFH5Ze84A](http://www.youtube.com/watch?v=C5LFH5Ze84A)
127) On Mars the acceleration due to gravity is \( \frac{12}{\text{sec}^2} \). (On Earth, gravity is much stronger at \( \frac{32}{\text{sec}^2} \).) In the movie, John Carter, it shows Carter leaping about 100 feet up on Mars. John Carter is an Earth man who has been transported to Mars so his leg muscles have been built to handle Earth's gravity while Mars gravity is a lot less. On Earth, Michael Jordan (a famous basketball player) had a vertical jump velocity of 16 ft/sec. Suppose John Carter could triple that initial jump velocity due to being on Mars, so his initial velocity would be \( v_0 = 48 \text{ ft/sec} \).

a) How high could he jump on Mars?

b) How long could he stay in the air before he hit the ground?

c) The movie shows Carter jumping about 100 ft. high. Is that about right by the Calculus?

d) What would his speed be when he hit the ground?

128) Referring to the previous problem, suppose the planet John Carter transported to had an acceleration due to gravity of \( \frac{2}{\text{sec}^2} \) and his initial jump velocity was still \( v_0 = 48 \text{ ft/sec} \).

a) How high could he jump under these conditions?

b) How long could he stay in the air before he hit the ground?

c) What would his speed be when he hit the ground?

Integration by Substitution – a more advanced Integration method to find antiderivatives by hand (algebraically)

129–R: Read and take notes on Section 7.1

129) Watch and take notes on this video. It explains and gives examples of integration by the substitution method: [http://youtu.be/x-svlZxucFE](http://youtu.be/x-svlZxucFE)

130) Write these notes on your paper:

**Justification of the substitution method:** Suppose \( F(x) \) is an antiderivative for \( f(x) \), so \( F'(x) = f(x) \). Then if we had a composition we would have \( (F(g(x)))' = F'(g(x)) \cdot g'(x) = f(g(x)) \cdot g'(x) \), (that last part of the chain of equals is because \( F'(x) = f(x) \) so \( F'(g(x)) = f(g(x)) \)). Hence, \( F(g(x)) \) is an antiderivative for \( f(g(x)) \cdot g'(x) \), that is, \( \int f(g(x)) \cdot g'(x) \, dx = F(g(x)) + C \). Now we use \( u \) to stand for \( g(x) \) and rewrite:

\[
\int f(u) \frac{du}{dx} \, dx = F(u) + C \quad \Rightarrow \quad \int f(u) \, du = F(u) + C
\]

**Know this justification for a test!**

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the integral using the given substitution.

131) \( \int x \cos (4x^2) \, dx \), \( u = 4x^2 \)  

A) \( \sin(4x^2) + C \)  

B) \( \frac{x^2}{2} \sin(4x^2) + C \) 

C) \( \frac{1}{8} \sin(4x^2) + C \)  

D) \( \frac{1}{u} \sin(u) + C \)
132) \[ \int \left[ 2 - \sin \frac{t}{2} \right]^2 \cos \frac{t}{2} \, dt, \quad u = 2 - \sin \frac{t}{2} \]

A) \( \frac{1}{3} \left( 2 - \sin \frac{t}{2} \right)^3 \) + C \\
B) \( \frac{2}{3} \left( 2 - \cos \frac{t}{2} \right)^3 \) + C \\
C) \( 2 \left( 2 - \sin \frac{t}{2} \right)^3 \) + C \\
D) \( -\frac{2}{3} \left( 2 - \sin \frac{t}{2} \right)^3 \) + C \\

133) \[ \int x^4(x^5 - 4)^5 \, dx, \quad u = x^5 - 4 \]

A) \( \frac{1}{6} (x^5 - 4)^6 \) + C \\
B) \( \frac{1}{30} x^{30} - 4 \) + C \\
C) \( \frac{1}{30} (x^5 - 4)^6 \) + C \\
D) \( \frac{1}{20} (x^5 - 4)^4 \) + C \\

134) \[ \int \frac{8s^3 \, ds}{\sqrt{8 - s^4}}, \quad u = 8 - s^4 \]

A) \(-4s^3 \sqrt{8 - s^4} + C\) \\
B) \(\frac{-2}{2\sqrt{8 - s^4}} + C\) \\
C) \(\frac{4s^4}{\sqrt{8 - s^4}}\) \\
D) \(-4\sqrt{8 - s^4} + C\) \\

135) \[ \int \frac{dx}{\sqrt{7x + 2}}, \quad u = 7x + 2 \]

A) \(\frac{7}{2} \sqrt{7x + 2} + C\) \\
B) \(2\sqrt{7x + 2} + C\) \\
C) \(\frac{2}{7} \sqrt{7x + 2} + C\) \\
D) \(\frac{1}{7(7x + 2)^{3/2}} + C\) \\

136) \[ \int 18(6x - 8)^{-6} \, dx, \quad u = 6x - 8 \]

A) \(-\frac{6}{5} (6x - 8)^{-5} + C\) \\
B) \(-\frac{3}{5} (6x - 8)^{-5} + C\) \\
C) \(-\frac{3}{7} (6x - 8)^{-7} + C\) \\
D) \((6x - 8)^{-5} + C\) \\

**ESSAY.**

137) **Watch and take notes on these videos.** They will help you with the next group of problems; **AND** they give instruction on how to **evaluate a definite integral using u-substitution**.

http://youtu.be/MtMAQlxQ7Zo, http://youtu.be/qkoDU3ktNsA

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

**Evaluate the integral.**

138) \[ \int \frac{x \, dx}{(7x^2 + 3)^5} \]

A) \(-\frac{7}{3} (7x^2 + 3)^{-4} + C\) \\
B) \(-\frac{1}{56} (7x^2 + 3)^{-4} + C\) \\
C) \(-\frac{7}{3} (7x^2 + 3)^{-6} + C\) \\
D) \(-\frac{1}{14} (7x^2 + 3)^{-6} + C\)
139) \[ \int \frac{\sin t}{(3 + \cos t)^4} \, dt \]

A) \( \frac{3}{(3 + \cos t)^3} + C \)  
B) \( \frac{1}{(3 + \cos t)^3} + C \)  
C) \( \frac{1}{5(3 + \cos t)^5} + C \)  
D) \( \frac{1}{3(3 + \cos t)^3} + C \)

140) \[ \int \frac{\ln x^7}{x} \, dx \]

A) \( \frac{1}{\ln x^7} + C \)  
B) \( \frac{1}{2}(\ln x^7)^2 + C \)  
C) \( \frac{1}{14}(\ln x^7)^2 + C \)  
D) \( \frac{1}{7}(\ln x^7)^2 + C \)

141) \[ \int \frac{1}{t^2} \sin\left(\frac{3}{t} + 5\right) \, dt \]

A) \( \frac{1}{3} \cos\left(\frac{3}{t} + 5\right) + C \)  
B) \( -\cos\left(\frac{3}{t} + 5\right) + C \)  
C) \( -\frac{1}{3} \cos\left(\frac{3}{t} + 5\right) + C \)  
D) \( 3 \cos\left(\frac{3}{t} + 5\right) + C \)

142) \[ \int \sin(8x - 4) \, dx \]

A) \( \frac{1}{8} \cos(8x - 4) + C \)  
B) \( 8 \cos(8x - 4) + C \)  
C) \( -\cos(8x - 4) + C \)  
D) \( -\frac{1}{8} \cos(8x - 4) + C \)

143) \[ \int \frac{dx}{x \ln x^3} \]

A) \( \ln x^3 + C \)  
B) \( \frac{1}{3} \ln(\ln x^3) + C \)  
C) \( \frac{1}{3} \ln x^3 + C \)  
D) \( \ln(\ln x^3) + C \)

144) \[ \int x^3 \sqrt{x^4 + 2} \, dx \]

A) \( \frac{2}{3}(x^4 + 2)^{3/2} + C \)  
B) \( -\frac{1}{2}(x^4 + 2)^{-1/2} + C \)  
C) \( \frac{1}{6}(x^4 + 2)^{3/2} + C \)  
D) \( \frac{8}{3}(x^4 + 2)^{3/2} + C \)

145) \[ \int x^5(x^6 - 6)^4 \, dx \]

A) \( [x^6 - 6]^5 + C \)  
B) \( \frac{[x^6 - 6]^3}{18} + C \)  
C) \( \frac{[x^6 - 6]^5}{6} + C \)  
D) \( \frac{[x^6 - 6]^5}{30} + C \)

ESSAY.

Solve the problem.

146) Evaluate \( \int x^2 \sqrt{5x^3 + 1} \, dx \).
147) Evaluate \( \int \frac{t^2}{\sqrt{16t^3 + 5}} \, dt \).

148) Evaluate \( \int \frac{(\sqrt{x} + 4)^3}{3\sqrt{x}} \, dx \).

149) Evaluate \( \int e^{3x} \, dx \).

150) Evaluate \( \int x^2 e^{x^3 + 1} \, dx \).

151) Evaluate \( \int \frac{1}{\sqrt{4-x}} \, dx \).

152) If we assume that wind resistance is proportional to velocity, then the downward velocity, \( v \), of a body of mass \( m \) falling vertically is given by \( v = \frac{mg}{k} \left( 1 - e^{-kt/m} \right) \), where \( g \) is the acceleration due to gravity and \( k \) is a constant. Find the height, \( h \), above the surface of the earth as a function of time. Assume the body starts at height \( h_0 \).

153) Over the past fifty years the carbon dioxide level in the atmosphere has increased. Carbon dioxide is believed to drive temperature, so predictions of future carbon dioxide levels are important. If \( C(t) \) is carbon dioxide level in parts per million (ppm) and \( t \) is time in years since 1950, three possible models are:

I \( C'(t) = 1.3 \)

II \( C'(t) = 0.5 + 0.03t \)

III \( C'(t) = 0.5e^{0.02t} \)

(a) Given that the carbon dioxide level was 311 ppm in 1950, find \( C(t) \) for each model.
(b) Find the carbon dioxide level in 2020 predicted by each model.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the substitution formula to evaluate the integral.

154) \( \int_0^1 \sqrt{x + 1} \, dx \) see this previous video for the u-substitution technique for a definite integral: http://youtu.be/MtMAO1xQ7Zo

A) \( \frac{4}{3} \sqrt{2} - \frac{2}{3} \)  
B) \( \frac{4}{3} \sqrt{2} \)  
C) \( 2\sqrt{2} - 1 \)  
D) \( 3\sqrt{2} - 3 \)

155) \( \int_{-1}^0 \frac{2t}{(2 + t^2)^{3/2}} \, dt \)

A) \( -\frac{5}{72} \)  
B) \( \frac{5}{72} \)  
C) \( -\frac{5}{18} \)  
D) \( -\frac{5}{36} \)
156) $\int_{-1}^{0} \frac{3t}{(4 + t^2)^4} \, dt$

A) $-\frac{61}{8000}$  
B) $\frac{61}{16000}$  
C) $-\frac{183}{8000}$  
D) $-\frac{61}{16000}$

157) $\int_{1}^{4} \frac{4 - \sqrt{x}}{\sqrt{x}} \, dx$

A) $-\frac{5}{2}$  
B) 5  
C) 10  
D) $\frac{5}{2}$

158) $\int_{0}^{\pi} (1 + \cos 5t)^2 \sin 5t \, dt$

A) $\frac{1}{15}$  
B) $\frac{8}{3}$  
C) $\frac{8}{15}$  
D) $\frac{1}{5}$

159) $\int_{0}^{\pi/2} \frac{\cos x}{(4 + 2 \sin x)^3} \, dx$

A) $-\frac{5}{288}$  
B) $\frac{5}{288}$  
C) $\frac{5}{576}$  
D) $-\frac{15}{64}$

160) $\int_{0}^{\pi/8} (1 + e^{\tan 2x}) \sec^2 2x \, dx$

A) $e$  
B) $-\frac{e}{2}$  
C) $2e$  
D) $\frac{e}{2}$

161) $\int_{\pi}^{3\pi/2} \frac{\sin \theta \, d\theta}{2 + \cos \theta}$

A) $-\ln 2$  
B) $\ln 3$  
C) 0  
D) $-\ln 3$

162) $\int_{0}^{3\pi/4} \frac{\tan \frac{x}{3}}{x} \, dx$

A) $\frac{3 \ln 2}{2}$  
B) $\frac{3\sqrt{2}}{2}$  
C) $-\frac{3 \ln 2}{2}$  
D) $-\frac{3\sqrt{2}}{2}$
163) \[ \int_{0}^{\ln \sqrt[4]{3/4}} \frac{4e^{4x}}{1 + e^{8x}} \, dx \]

A) \( \frac{\pi}{12} \)  
B) \( -\frac{\pi}{6} \)  
C) \( -\frac{\pi}{12} \)  
D) \( \frac{\pi}{6} \)

ESSAY.

164) Evaluate \( \int_{0}^{\pi/2} e^{-\cos(\theta)} \sin(\theta) \, d\theta \)

165) With \( t \) in years since 2000, the population, \( P \), of the world in billions can be modeled by \( P = 6.1e^{0.012t} \).
   (a) What does this model predict for the world population in 2010? In 2020?
   (b) Use the Fundamental Theorem to predict the average population of the world between 2000 and 2010.

166) Find the integrals for 71–77 odd. For these, make the usual \( u \) substitution but then rewrite the remaining original variable in terms of \( u \) also. (These problems are from Section 7.1)

71. \[ \int y \sqrt{y + 1} \, dy \]

73. \[ \int \frac{t^2 + t}{\sqrt{t + 1}} \, dt \]

75. \[ \int x^2 \sqrt{x - 2} \, dx \]

77. \[ \int \frac{t}{\sqrt{t + 1}} \, dt \]

167) Do Review Exercises and Problems for Chapter 5 #1 – 65 EVERY OTHER ODD, and Review Exercises and Problems for Chapter 6 #1 – 33 EVERY OTHER ODD, #49 – 65 EVERY OTHER ODD, #85, 89.
Answer Key
Testname: INTEGRATION WORKSHEET 1 - UNDERSTANDING THE DEFINITE INTEGRAL

1) B
2) A
3) A
4) B
5) D
6) A
7) A
8) B
9) A
10) C
11) C
12)
13)
14) C
15) D
16) C
17) D
18) C
19) B
20) The applet shows the answer
21) The applet shows the answers
22) The applet shows the answers.
23) see applet
24)
25) (a) Left sum  (b) Upper estimate  (c) 6  (d) Δt = 2  (e) Upper estimate ≈ 24.4 (eyeball the rectangle heights and get close)
26) Between 140 and 150 meters. On this one, a quick way to get an estimate is to see that the triangle with vertices (0,0), (6,40), and (6,0) has an area in the ballpark of the area under the curve, and then find that area. It is sort of close but is an under estimate. Area of triangle is (1/2)(6)(40) = 120
27) 9) 0 cm, no change in position
       11) 15 cm to the left
28) Lower: 14.73 minutes   Upper: 15.93 minutes
29) (a) 3 ≤ t ≤ 5, 9 ≤ t ≤ 10  (b) 3600 ft east  (c) At t = 8 minutes  (d) 30 seconds longer
30) (a) Use your calculator  (b) 3 sec, 144 feet  (c) 80 feet
31) D
32) C
33)
34)
35)
36)
37)
38)
39)
40) (a) Left; smaller  (b) a, b, n and Δx are 0, 2, 6, 1/3 respectively
41) 5) 28.5   7) 1.4936   9) 0.3103
42) 60 (approx.)
43) −40 (approx.)
44) a) 13   (b) 1
45) (a) 16.25  (b) 15.75  (c) No , since the curve has parts above and below the x-axis. But \( \int_{0}^{3} |f(x)| \, dx \), which you would use \( y_1 = \text{abs}(x^3 - x) \) to use with the integral program to evaluate, IS the same as part (a) !

46) 23) 1  25) 1.977  27) 24.694  29) (a) 13  (b) -2  (c) 11  (d) 15  31) (a) -2  (b) -A/2

50) A  51) A  52) D

55) a) 8  b) \( F(x) = x^2 \) (actually \( x^2 + \text{any constant} \) would work also)  c) \( F(3) - F(1) = 3^2 - 1^2 = 8 \)
   d) It sure does check!

56) a) \( \int_{0}^{8} 32e^{0.05t} \, dt \) billion barrels  b) The antiderivative is \( 640e^{0.05t} \) from wolframalpha. Evaluating at 8 and subtracting the evaluation at 0 we get 314.1 billion barrels  c) 314.8 billion barrels (the .7 difference is because, as we have seen, n=100 still creates a bit of error in the sum.)

57) a) 346.6 hours  b) \( \int_{0}^{346.6} 2.4e^{-0.004t} \, dt \) millirems  c) 450.6 millirems  d) 450.016 millirems

58) a) \( \int_{1}^{2} (\cos(t)+5) \, dt \) miles  b) 5.07 miles  c) 5.068 miles

59) a) \( \int_{0}^{2} R(t) \, dt \) b) about 2.8 to 3.4 gallons

60) a) \( \int_{0}^{5} f(t) \, dt \) b) 177.27 billion barrels

61) $2392.80  Set up \( \int_{0}^{52} 3.75e^{-0.008t} \, dt \) , evaluate n=200, TRAP, get 159.52 tons, then multiply by $15 per ton and get $2392.80 . Note: Wise to use a higher n size since the integral goes from 0 to 52 (a larger interval).

62) a) .6827 = 68.27% are average  b) .0214 = 2.14% are gifted  c) .0013 = 0.13% only 13 out of 10,000 are genius!
   (FYI: the “bright” category is from \( z=1 \) to \( z=2 \) for 13.59%)

63) 45.8 degrees C.  Set up \( \int_{0}^{10} -7e^{-0.1t} \, dt \) , eval with n=100, TRAP get \( \approx 44.25 \) degrees C, BUT that is how much the temperature has CHANGED over the 10 minutes. The coffee was at 90 C at time \( t = 0 \), so by the 10 minute mark the coffee is at \( 90 - 44.25 = 45.8 \) degrees C

64) a) your guess  b) 1.3333333  c) \( \frac{4}{3} \) yes it matched exactly.

65)

66) C
Answer Key
Testname: INTEGRATION WORKSHEET 1 - UNDERSTANDING THE DEFINITE INTEGRAL

67) A
68) D
69) $6080
70) (a) 8.5     (b) 1.7
71)
72) D
73) B
74) B
75) C
76) C
77) A
78) C
79) D
80) B
81) A
82) D
83) C
84) B
85) B
86)
87) answers:

5

\[ F(0) = 1 \]
\[ F(0) = 0 \]

7

\[ F(0) = 1 \]
\[ F(0) = 0 \]

9

\[ F(0) = 1 \]
\[ F(0) = 0 \]

11

\[ F(0) = 1 \]
\[ F(0) = 0 \]

88) 128, 169, 217 approx.

89) answer:

\[ y \]
\[ 2 \]
\[ t_1 \ t_2 \ t_3 \ t_4 \ t_5 \]
Critical points: (0,5), (2,21), (4,13), (5,15)

(a) 0; 3,000; 12,000; 21,000; 27,000; 30,000  
(b)

answers:
1. $5x$
2. $x^3/3$
3. $2z^{3/2}/3$
4. $-1/t$
5. $-1/(2z^2)$
6. $e^z$
7. $2t^3/3 + 3t^4/4 + 4t^5/5$
8. $t^2/2 + \ln|t|$
96) answers:
17. \(3t^2 + C\)
19. \(x^3/3 - 2x^2 + 7x + C\)
21. \(t^4/4 + 5t^2/2 - t + C\)
23. \(-\cos x + \sin x + C\)
25. \(2t^{1/2} + C\)
27. \(-5/(2x^2) + C\)

97) answers:
37. \((5/2)x^2 + 7x + C\)
39. \(2t + \sin t + C\)
41. \(3e^x - 2\cos x + C\)
43. \((5/3)x^3 + (4/3)x^{3/2} + C\)
45. \(16\sqrt{x} + C\)
47. \(e^x + 5x + C\)
49. \((2/5)x^{5/2} - 2\ln |x| + C\)

98) D
99) D
100) C
101) C
102) 51.) 36 53.) \((-\sqrt{2}/2) + 1 = 0.293\) 55.) 3e2 - 3 = 19.167 57.) 1 - \cos 1 \approx 0.460 59.) 16/3 \approx 5.333
103.) 5.5 ft
104.) 9
105.) A
106.) B
107.) B
108.) A
109.) A
110.) A
111.) B
112.)
113.) 13.) 4.389 15.) 0.083 17.) 7.799 19.) 0.172
114.)
115.)
116.) discuss in class.
117.) \(\int_0^1 2t \, dt = 0, \int_0^2 2t \, dt = 1, \int_0^3 2t \, dt = 4, \int_0^4 2t \, dt = 9, \int_0^4 2t \, dt = 16\) b) Yes, the graph is \(x^2\) and that is an antiderivative of \(2x\).
118.) see video
119) 

120) \( f(x) = 5 + \int_{1}^{x} \frac{\sin(t)}{t} \, dt \) notice how when \( x = 1 \) the integral is \( \int_{1}^{1} \frac{\sin(t)}{t} \, dt = 0 \) and then you add the 5 to get 5 so that \( f(1) = 5 \).

121) 11) \( \cos(x^2) \) 13) \( (1+x)^{200} \) 15) \( \arctan(x^2) \) The integral statements are constructed antiderivatives of the integrand functions. So when you take the derivative of these integrals, of course you get back to the integrand functions.

122) answers:

\[ 35 \quad 2x \ln(1 + x^4) \]

\[ 37 \quad -2 \sin(\sqrt{2t}) \]

123)

124) a) \( h(t) = -\frac{5}{2}t^2 + 10t \)  b) \( v(t) = -5t + 10 = 0 \) \( \Rightarrow \) \( t = 2 \) sec he achieves max height. Max height = \( h(2) = 10 \) ft.  c) \( h(t) = 0 \) at \( t = 0 \) and \( t = 4 \) sec. so he was off the ground for 4 sec.

125) \( h(t) = -16t^2 - 20t + 250 \)

126)

127) a) \( s = 96 \) ft. max height  b) 8 seconds in the air.  c) Yes, the movie was in line with the Calculus!!  d) \(-48 \) ft/sec , that's \( 48 \) ft/sec downward = \( 32.7 \) miles/hour

128) a) \( s = 576 \) ft. max height  b) 48 seconds in the air  c) \(-48 \) ft/sec , that's \( 48 \) ft/sec downward = \( 32.7 \) miles/hour

129)

130)

131) C

132) D

133) C

134) D

135) C

136) B

137)

138) B

139) D

140) C

141) A

142) D

143) B

144) C

145) D

146) \( \frac{2}{45} (5x^3 + 1)^{3/2} + C \)

147) \( \frac{\sqrt{16t^3 + 5}}{24} + C \)

148) \( \frac{(\sqrt{x} + 4)^4}{6} + C \)
149) \( \frac{1}{3}e^{3x} + C \)  
But \( \int e^{ax} \, dx \) doesn't really have to be done with substitution, we just know that \( \int e^{ax} \, dx = \frac{1}{a} e^{ax} + C \)  

150) \( \frac{1}{3}e^{x^3+1} + C \)  
151) \(-2\sqrt{4-x} + C\)  
152) \( \frac{mg}{k} t - (\frac{m^2g}{k^2})(1 - e^{-kt/m}) + h_0 \)  

153) (a) I \( C(t) = 1.3t + 311 \), II \( C(t) = 0.5t + 0.015t^2 + 311 \), III \( C(t) = 25e^{0.02t} + 286 \)  
(The 286 is obtained this way: The integral answer is \( C(t) = 25e^{0.02t} + C \) so we have to find C. At \( t=0 \) we have \( C(0) = 311 \) ppm, so \( 25e^{0.02(0)} + C = 311 \) \( \Rightarrow 25 + C = 311 \) \( \Rightarrow C = 311 - 25 = 286 \)  

(b) I 402 ppm, II 419.5 ppm, III 387.380 ppm  
154) A  
155) A  
156) D  
157) B  
158) C  
159) C  
160) D  
161) A  
162) A  
163) A  
164) 1 - (1/e)  
165) (a) 6.9 billion, 7.8 billion (b) 6.5 billion  
166) answers:  

\[
\begin{align*}
71 \quad & (2/5)(y + 1)^{5/2} - (2/3)(y + 1)^{3/2} + C \\
73 \quad & (2/5)(t + 1)^{5/2} - (2/3)(t + 1)^{3/2} + C \\
75 \quad & (2/7)(x - 2)^{7/2} + (8/5)(x - 2)^{5/2} + (8/3)(x - 2)^{3/2} + C \\
77 \quad & (2/3)(t + 1)^{3/2} - 2(t + 1)^{1/2} + C 
\end{align*}
\]