Pierce College
Math Department

Materials and Resources
for
Math 115 Instructors

Fall Semester, 2011
Welcome to the Developmental Math team at Pierce College. We are in the process of redesigning and coordinating our developmental math program, and we look forward to working with all of you. We hope that you will work with us to provide our students with a sequence of courses that is coherent, relevant, and carefully coordinated.

Adjunct instructors are an important part of our faculty, and we encourage you to participate in departmental activities and professional development. In case you have any questions or concerns about your course at Pierce, we are assigning each of you a liaison from the full-time faculty. Please do not hesitate to contact your liaison if we can be of assistance.

**Fall 2011 Math 115 Faculty**

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Here is a link to our department website, where we will post this document and additional resources as they are developed:

http://faculty.piercecollege.edu/yoshibw/chair/
**Course Content for Math 115**

Because passing Math 125 is now a requirement for an AA degree, we no longer need to cram a lot of topics into Math 115 that will only need to be reviewed in Math 125. Instead, we can concentrate on the core elementary topics, strive for understanding of concepts, and try to address some of the more egregious deficiencies in the preparation of our Math 125 students.

The outline below shows the current language of the COR followed by proposals for more careful delineation of content.

1. **Real numbers**
   a. Signed numbers
   b. Order of operations
   c. Absolute value

1. **Number properties:** Discussion of how properties apply to algebraic expressions.
   **Ex:** Is $2ab$ equivalent to $2ba$? Is $2ab$ equivalent to $2a \cdot 2b$?
   Formal properties of the real numbers in abstract terms, e.g. $a(bc) = (ab)c$, are inappropriate at this level; they are a topic for precalculus.
   a. Signed numbers: students should know what integers are, but should not have to identify rational and irrational numbers. Instead, they should learn the difference between an exact value and a decimal approximation, e.g. $\frac{2}{3} \neq 0.6$
   b. Order of operations: operations on signed numbers is part of Math 110, and could be reviewed briefly in Math 115 as part of order of operations. Complexity of problems should be kept to a reasonable level.
   **Ex:** Simplify: $12 - 8(-6)^2$
   c. Absolute value should not be included in Math 115. In the statement of the rules for operations on signed numbers, we can refer to the "unsigned part" of a number.

2. **Algebraic expressions**
   a. Simplifying expressions
   b. Evaluating by substitution

2. **Algebraic expressions:** writing algebraic expressions with one or two operations to model situations should be a major part of Math 115.
   **Ex:** Wendy’s income tax is $1200 more than 6% of her adjusted income.
   a. Simplifying expressions: keep to the level that might be encountered in applications.
   **Ex:** $3x - 2(4x - 5)$
   b. Evaluating by substitution: again, keep to a reasonable level

3. **Equations and inequalities**
   a. Solving linear equations and inequalities
   b. Modeling with equations

3. **Equations and inequalities**
   a. Solving: include both common fractions and decimal fractions
   b. Modeling: Concentrate on problems of the form $L(x) = k$, where $L(x)$ is a linear model, or problems from geometry (especially area, perimeter, and volume), or using formulas. (Most other standard word problems are better suited to solution by a system of equations.) Avoid artificial word problems such as coin and age problems.
   **Ex:** The Melrose Theater Company sold tickets to a matinee for $16 each. The play cost them $650 to stage, and they made $918 profit. How many tickets did they sell?
4. Polynomials and exponents
   a. Laws of exponents
   b. Negative exponents
   c. Scientific notation
   d. Addition and multiplication of polynomials

4. Expressions with exponents: study of polynomials per se are not appropriate for Math 115; they should be introduced in Math 125.
   a. Laws of exponents: simple examples only.
      Ex: Simplify \((2a^2)^3 \div 4a^3\)
   b. Negative exponents: definitions and simple exercises only.
      Ex: Simplify and write without negative exponents: \(\frac{3x^{-3}}{9x^9}\)
   c. Scientific notation: include using a calculator
   d. Addition and multiplication: No polynomial division.
      Ex: Simplify \((5x^3 - 2x^2 + 4) - (6x^3 - 3x^2 - 3x)\)

5. Rational expressions
   a. Operations with rational expressions
   b. Equations involving rational expressions
   c. Ratio and proportion

5. Algebraic fractions: concentrate on modeling and evaluation
   Ex: Morgan drove across country in \(h\) hours, but he estimates that he spent 10 hours stopped for rest and meals.
      a. If he drove a total of 2800 miles, what was his average speed on the road?
      b. Evaluate your fraction for \(h = 50\). What does your answer mean in this context?
   a. Operations: linear denominators only. No complex fractions.
      Ex: Subtract: \(\frac{x}{x + 1} - \frac{2}{x - 2}\)
   b. Equations: simple denominators only. Concentrate on equations of the form \(\frac{\text{(expression in } x)}{k} = c\).
      Ex: Solve \(2.5 = \frac{300}{10 - y}\)
   c. Ratio and proportion: another major topic for Math 115. Students should be able to identify proportional variables from a table, a graph, and an equation. Scaling and similarity of plane figures should be included.
      Ex: Decide whether the two variables are proportional:

<table>
<thead>
<tr>
<th>(x)</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>16</th>
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<tr>
<td>(y)</td>
<td>3.2</td>
<td>8</td>
<td>16</td>
<td>25.6</td>
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</tbody>
</table>

6. Linear equations in two variables
   a. Slope
   b. Slope-intercept and point-slope forms
   c. Graphing, reading graphs

6. Linear equations in two variables: students should be able to write and graph linear models.
   Ex: The water in Silver Pond is 10 feet deep, but the water level is dropping at a rate of \(\frac{1}{2}\) inch per week. Write an equation for the depth, \(d\), of the pond after \(w\) weeks. Graph your equation.
a. Slope: students should calculate slope from a graph and interpret as a rate of change.

**Ex:** The graph shows the distance traveled by a driver for a cross-country trucking firm in terms of the number of hours she has been on the road. Compute the slope of the graph. What is the meaning of the slope for the problem?

b. Slope-intercept and point-slope forms: including models in context

**Ex:** The height of a string bean vine each day is given in inches by \( h = 18 + 3d \), where \( d = 0 \) represents today. Graph the line by the slope-intercept method. Explain what the slope and the \( y \)-intercept tell us about the problem.

c. Graphing, reading graphs: students should graph by both the intercept method and the slope-intercept method. They should interpret points on a graph, including the intercepts. Equations of parallel and perpendicular lines should not be included.

**Ex:** The temperature in Nome was \(-12^\circ\) at noon and has been rising at a rate of \(2^\circ\) per hour all day. Find the intercepts of the graph and interpret their meaning in the context of the problem situation.

7. Systems of equations
   a. Solution by graphing
   b. Substitution method
   c. Elimination method
   d. Word problems

7. Systems of equations
   a. Solution by graphing
   b. Substitution method
   c. Elimination method
   d. Word problems: As noted above under section 3, many standard word problems are better suited to systems of equations, including motion and mixture problems. Mixture problems should be taught as weighted averages, with suitable numerical examples before algebraic problems.

**Ex:** The chemistry department has 80 students, of whom 35% are women. The physics department has 60 students, of whom 15% are women. (a) How many chemistry students are women? How many physics students are women? (b) How many students are there in both departments? How many of them are women? (c) What percent of the students in chemistry and physics are women?

8. Factoring techniques
   a. Common factors
   b. Difference of squares
   c. Quadratic trinomials

   a. Common factors
   **Ex:** Factor \( 3b^4 + 12b^3 + 12b^2 \)
   b. Difference of squares
   c. Quadratic trinomials: monic or prime quadratic coefficient only
   **Ex:** Factor \( 2x^2 + x - 3 \)

9. Quadratic equations
   a. Solving quadratic equations by factoring and by the quadratic formula
   b. Graphing simple parabolas
   c. Pythagorean theorem
9. Quadratic equations: simple applications only, such as falling objects
   a. Solving by factoring and quadratic formula: and extracting roots. No completing the square.
   b. Graphing simple parabolas: find $x$-intercepts and vertex
   Ex: $y = x^2 - 2x - 8$
   c. Pythagorean theorem (but no distance and midpoint formulas)

10. Roots and radicals
   a. Simplifying radical expressions
   b. Extraction of roots

   10. Roots and radicals
   a. Simplifying radical expressions: e.g. $\sqrt{12} = 2\sqrt{3}$. Simple operations only, e.g. $\sqrt{x} + 2\sqrt{x} = 3\sqrt{x}$, $\frac{7}{\sqrt{7}} = \sqrt{7}$. (This last should be accomplished by the definition of square root, not by rationalizing the denominator.)
   b. Extraction of roots: include simple cubic equations and cube roots.
   Ex: Solve $3\sqrt{5x - 1} - 5 = 16$; solve $\frac{3}{4} \pi r^3 = 26$; solve $20 - 3\sqrt{x} = 14$

In particular, the complexity of problems in the following topics should not exceed the minimum necessary to assess comprehension:
1. Factoring
2. Laws of exponents
3. Simplifying radicals
4. Operations on algebraic fractions

Also, although the notion of function is implicit in much of the work with linear models, explicit study of functions and function notation is not included in Math 115. Functions are introduced in Math 125.

Our list of outcomes has minor changes:

Upon successful completion of this course, the student will be able to:
1. Calculate with signed numbers, order of operations Omit: "and absolute value"
2. Perform operations with algebraic expressions
3. Model problems and solve linear equations and inequalities in one variable
4. Write ratios and solve proportions
5. Solve formulas for one variable
6. Factor polynomials
7. Add and multiply polynomials
8. Simplify expressions involving exponents
9. Convert between standard and scientific notation
10. Perform operations with algebraic fractions
11. Solve equations involving algebraic fractions
12. Model and solve quadratic equations in one variable
13. Apply the Pythagorean theorem to solve problems
14. Graph simple parabolas
15. Model and solve a 2x2 system of linear equations
16. Graph linear equations (Omit: and inequalities) in two variables
17. Read and interpret information from graphs
18. Calculate and interpret slope
19. Perform operations with radicals
Kaufmann, **Elementary Algebra**

Omit the following topics and sections:
1. In Section 1.1, omit sets and set notation
2. Sections 1.3 and 1.4 (review material)
3. In Section 2.3, omit subsets and field axioms of the real numbers
4. In Section 3.5, omit set builder notation
5. Section 5.5
6. In Section 6.4, factor quadratic trinomials with prime lead coefficient only
7. In Chapter 7, limit the complexity of exercises as follows:
   7.1 omit #29-60
   7.2 omit #29-46
   7.4 omit #25-60
   7.6 omit #19-32
8. In Section 7.4, omit complex fractions
9. In Section 8.2, omit graphing linear inequalities in two variables
10. In Section 8.4, omit parallel and perpendicular lines
11. In Section 9.4, omit rationalizing binomial denominators
12. Omit Section 10.2
13. Omit Chapter 11

This text needs supplementing on writing and graphing linear models, interpreting intercepts and slope, solving equations graphically, and reading and interpreting information from graphs.

Rockswold, **Beginning Algebra**

Omit the following topics and sections:
1. Section 1.2
2. In Section 1.4, omit, real and irrational numbers and absolute value
3. Sections 1.5 and 1.6 (review material)
4. Section 1.7
5. Section 3.1 (review material)
6. In Section 3.5, omit parallel and perpendicular lines
7. Omit Section 4.4
8. In Section 5.6, omit division by a polynomial
9. In Section 6.3, factor quadratic trinomials with prime lead coefficient only
10. Section 6.5
11. Section 6.7
12. In Chapter 7, limit the complexity of exercises as follows:
   7.2: omit #49-74
13. Omit Section 7.5
14. In Section 7.7, omit inverse variation
15. In Section 8.4, omit rationalizing binomial denominators
16. Section 8.6
17. Section 9.3
18. In Section 9.4, omit the discriminant
19. In Section 9.5, omit operations on complex numbers
20. Section 9.6

This text often uses false "models" based on curve-fitting in its application problems. Please try to avoid these problems.
Sullivan, Elementary Algebra

Omit the following topics and sections:
1. Sections 1.1-1.6
2. In Chapter 2, omit set notation and set builder notation
3. Section 3.6
4. Section 3.7
5. Section 4.6
6. In Section 5.5, omit division by a binomial
7. In Section 5.6, factor quadratic trinomials with prime lead coefficient only
8. In Chapter 7, limit the complexity of exercises as follows:
   7.1: omit #61-76
   7.2: omit #45-66
   7.5: omit #67-78
9. Section 7.6
10. In Section 7.9, omit inverse variation
11. In Section 8.1, omit classification of radicals
12. In Section 8.5, omit rationalizing binomial denominators
13. In Section 8.6, omit equations with two radicals
14. In Section 8.7, omit rational exponents
15. Section 9.2
16. In Section 9.3, omit the discriminant
17. Section 9.5
18. Chapter 10

Yoshiwara, Introductory Algebra

Omit the following topics and sections:
1. Sections 1.1 and 1.2
2. Sections 2.1-2.3 (review material)
3. Section 2.10
4. Section 3.2
5. Section 4.3
6. In Section 4.9, factor quadratic trinomials with prime lead coefficient only
7. Section 5.5
8. In Section 5.8, omit complex fractions
Best Teaching Practices

1. Lesson Design
   • When preparing a lesson, identify the 3 or 4 most important points and concentrate on those.
   • Whenever possible, provide an activity that demonstrates the lesson. Prepare a worksheet or other materials to keep students engaged.
   • Use multiple representations: tables, graphs, equations.
   • Choose examples carefully, to illustrate the ideas without unnecessary distractions or complicated calculations.

2. Content Guidelines
   • Do not overemphasize skills at the expense of concepts.
   • Avoid unnatural computational complexity. (If you can’t think of any reasonable elementary application, it is too complex.)
   • Include frequent practice writing algebraic expressions, gradually increasing in difficulty.
   • Whenever possible, use realistic applications. Avoid contrived applications.

3. In the Classroom
   • Alternate short (5 or 10-minute) lecture periods with group work or other activities.
   • Do not write definitions, theorems, or other parts of the textbook on the board. Instead, refer students to the textbook.
   • Make connections explicit: to previous material, between new ideas, between principles and application.
   • Ask questions that are not simply computational or algorithmic.
   • Provide a summary and wrap-up at the end of class.

4. Student Work
   • Require some written homework, and provide feedback on presentation: organization, use of notation, etc.
   • Give frequent short quizzes.
   • Require students to read the textbook (and provide incentive to do so).
   • Design tests, quizzes, and homework assignments to reflect the goals and SLO’s of the course.
Sample Problems

In Math 115, we would like to place more emphasis on writing algebraic expressions and equations and interpreting graphs, and less emphasis on the complexity of calculation.

Properties of Numbers
1. True or False:
   a. \( \frac{2}{3} = 0.6 \)
   b. \( \frac{2}{3} = 0.66 \)
   c. \( \frac{3}{5} = 0.6 \)
   d. \( \frac{2}{3} = 0.6667 \)

2. a. Use your calculator to verify that
   \[ \frac{-2}{5} = \frac{-2}{5} = \frac{2}{-5} \]
   b. Does \( \frac{-2}{5} = \frac{-2}{-5} \)?

3. Evaluate for \( x = 5 \). What do you notice?
   a. \( \frac{-3}{4}x \)
   b. \( \frac{-3x}{4} \)
   c. \( -0.75x \)

4. Does \( \frac{-8}{5}x = \frac{-8}{5x} \)? Support your answer with examples.

5. Simplify each product. Which product in each pair requires the distributive law?
   a. \( 8(4c) \)
   b. \( 8(4 + c) \)
   c. \( -6(3 + m) \)
   d. \( -6(3m) \)

7. Simplify each expression if possible. Then evaluate for \( x = 3,\ y = 9 \).
   a. \( 2(xy) \)
   b. \( 2(x + y) \)
   c. \( 2 - xy \)
   d. \( -2xy \)

8. Which of the following is a correct application of the distributive law?
   a. \( 5(3a) = 15a \)
   b. \( 5(3 + a) = 15 + a \)
   c. \( 5(3a) = 15(5a) \)
   d. \( 5(3 + a) = 15 + 5a \)

I Linear Equations

Write an Algebraic Expression
1. Choose a variable for the unknown quantity and write an algebraic expression.
   (One operation)
   9. The radius of the circle increased by 5 centimeters
   10. Three-fifths of the savings account balance
   11. 49% of the total
   12. The perimeter of the triangle diminished by 10 feet
   13. The ratio of 25 to the number of professors
   (Two operations)
   14. Three inches less than twice the width
   15. Twenty dollars more than 40% of the principal
   16. Fifteen miles less than 1.6 times the distance to the city
   17. One-third of the total number of cars and trucks
   18. Twelve times the sum of rent and utilities
   19. The ratio of hits plus walks to times at bat
   20. 6% of the sum of your regular salary and your overtime pay.
For Problems 21-23, write an algebraic expression for the area or perimeter of the figure. Include units in your answers. The dimensions are given in inches.

\[ \text{area} = \quad \text{perimeter} = \quad \text{area} = \]

24. a. Daniel and Lara together made $480. If Daniel made \( x \) dollars, how much did Lara make?
   b. Alix spent $500 on tuition and books. If she spent \( x \) dollars on books, how much was her tuition?
   c. Thirty children signed up for summer camp. If \( x \) boys signed up, how many girls signed up?

For Problems 25-26, identify the unknown quantity, assign a variable and write an algebraic expression.

25. Hugo's Auto Shop paid $4000 in expenses this month. Write an expression for their profit in terms of their revenue.

26. Eggnog is 70% milk. Write an expression for the amount of eggnog in a container of milk.

27. The width of a rectangle is 4 inches longer than half the length. Write expressions in terms of \( L \), the length of the rectangle.
   a. The width of the rectangle
   b. The perimeter of the rectangle
   c. The area of the rectangle

28. Two slices of pizza and a soda contain 678 calories. Write expressions in terms of \( x \), the number of calories in a soda.
   a. The number of calories in a slice of pizza
   b. The number of calories in two sodas
   c. The number of calories in three slices of pizza and two sodas

29. Albert and Isaac left the same hotel at the same time and drove for 3 hours, but Albert drove 15 mph faster than Isaac. Write expressions in terms of Isaac's speed, \( s \).
   a. How far Isaac drove
   b. How far Albert drove
   c. If they drove in the same direction, how far apart they are now
   d. If they drove in opposite directions, how far apart they are now

30. A box of AlmondOats contains 15 ounces of cereal made of oat flakes and sliced almonds. Oat flakes cost 15 cents per ounce, and almonds cost 35 cents per ounce. Write expressions in terms of \( a \), the number of ounces of almonds in the box.
   a. The number of ounces of oat flakes in the box
   b. The cost of the almonds
   c. The cost of the oat flakes
   d. The total cost of a box of AlmondOats
Write an Equation
31. Choose the appropriate equation to model each problem.

\[
\begin{align*}
x + 7 &= 26 & 7x &= 6 & \frac{x}{7} &= 26 \\
x - 7 &= 26 & \frac{x}{26} &= 7 & \frac{26}{x} &= 7
\end{align*}
\]

a. Sarah drove 7 miles farther to her high school reunion than Jenni drove. If Sarah drove 26 miles, how far did Jenni drive?
b. Lurline and Rozik live 26 miles apart. They meet at a theme park between their homes. If Lurline drove 7 miles to the park, how far did Rozik drive?
c. Doris is training for a triathlon. This week she averaged 26 miles per day on her bicycle. If she rode every day, what was her total mileage?
d. Glynnis jogged the same route every day this week for a total of 26 miles. How long is her route?
e. Astrid divided her supply of colored pencils among the 26 children in her class, and each child got 7 pencils. How many pencils does she have?
f. Ariel lost 7 of the beads on her necklace, and now there are 26. How many were there originally?

32. Find the pattern and fill in each table. Then write an equation for the second variable in terms of the first variable.

<table>
<thead>
<tr>
<th>a. m</th>
<th>b. h</th>
<th>c. t</th>
<th>d. n</th>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
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<tr>
<td>m</td>
<td>h</td>
<td>t</td>
<td>n</td>
</tr>
</tbody>
</table>

33. Danny weighs 32 pounds more than Brenda. If Danny weighs 157 pounds, how much does Brenda weigh?

a. What are we asked to find? Choose a variable to represent it.
b. Find two ways to express Danny's weight, and write an equation.
c. Solve the equation and answer the question in the problem.

34. Miranda worked 20 hours this week and made $136. What is Miranda's hourly wage?

a. What are we asked to find? Choose a variable to represent it.
b. Find two ways to express Miranda's total earnings, and write an equation.
c. Solve the equation and answer the question in the problem.

35. Clive loaned his brother some money to buy a new truck, and his brother agreed to repay the loan in 1 year with 3% interest. Clive earned $75 interest on the loan. How much did Clive loan his brother?

36. Emily spends 40% of her monthly income on rent. If her rent is $360 a month, how much does Emily make?
Solve a Linear Equation or Inequality

- Solve.
  37. \( x - 9 = -4 \)
  38. \( -9z = 12 \)
  39. \( -3w = -15 \)
  40. \( 36 = \frac{-3b}{5} \)
  41. \( -3 - x = -5 \)
  42. \( 3c - 7 = -13 \)
  43. \( -5 = -2 - 3t \)
  44. \( 1 - \frac{b}{3} = -5 \)
  45. \( \frac{3y}{5} + 2 = -4 \)

- Solve each inequality, and graph the solutions on a number line.
  46. \( 2x + 3 > 7 \)
  47. \( -3x + 2 \leq 11 \)
  48. \( -3 > \frac{2x}{3} + 1 \)
  49. \( -3 \leq 3x \leq 12 \)
  50. \( -6 > 4 - 5b > -21 \)
  51. \( -8 \leq \frac{5w + 3}{4} < -3 \)

- Solve each equation or inequality.
  52. \( 15 - 9t = 33 - 5t \)
  53. \( -6s = 3s \)
  54. \( 3x + 5 > 2x + 3 \)
  55. \( -8g + 35 = 9g - 13 + g \)
  56. \( -15y + 5 - 2y - 4 \geq -12y + 21 \)
  57. \( 6(3y - 4) = -60 \)
  58. \( 5w - 64 = -2(3w - 1) \)
  59. \( -22c + 5(3c + 4) = 20 + 8c \)
  60. \( 4 - 3(2t - 4) > -2(4 - 3t) \)
  61. \( 0.25(x + 3) - 0.45(x - 3) = 0.30 \)

Solve Word Problems

- Solve each problem in three steps:
  a. Choose a variable for the unknown quantity.
  b. Write an equation that involves your variable.
  c. Solve your equation, and answer the question.

  62. Simona bought a car for $10,200. She paid $1200 down and will pay the rest in 36 monthly installments. How much is each installment?
      (Hint: Write an equation about the total amount Simona pays.)

  63. For English class, Earlene has ten days to read A Tale of Two Cities, which is 417 pages long. If she reads 75 pages the first day, how many pages must she read on each of the remaining 9 days?
      (Hint: Write an equation about the total number of pages Earlene must read.)

  64. The Tree People planted new trees in an area that was burned by brush fires. 60% of the seeds sprouted, but gophers ate 38 of the new sprouts. That left 112 new saplings. How many seeds were planted?
      (Hint: Write an equation about the number of seeds that survived.)

- Find the value of \( x \) in each figure.
  65. \[
  \begin{array}{c}
  \text{Area} = 20
  \\
  3
  \end{array}
  \]

  66. \[
  \begin{array}{c}
  \text{Area} = 70
  \\
  5
  \end{array}
  \]

  67. An apple and a glass of milk together contain 260 calories.
      a. If an apple contains \( a \) calories, how many calories are in a glass of milk?
      b. Write an expression for the number of calories in two apples and three glasses of milk.
      c. If two apples and three glasses of milk contain 660 calories, find the number of calories in an apple.
68. Last year, Delbert and Francine together made $35,000.
   a. If Delbert’s salary was \( \frac{x}{2} \), what was Francine’s salary?
   b. This year, Delbert got a 5% raise and Francine got a 6% raise. Write an expression for their total increase in income.
   c. This year, Delbert and Francine together made $36,950. What was Delbert’s salary last year?

69. The length of a rectangular vegetable garden is 6 yards more than twice its width.
   a. Write an expression for the perimeter of the garden in terms of its width.
   b. Ann bought 42 yards of fence to enclose the garden. What are its dimensions?

70. Melody sold 47 tickets to a charity concert. Reserved seats cost $10 and open seating was $6 a ticket. Let \( x \) represent the number of reserved seats she sold. Write expressions in terms of \( x \) for:
   a. The number of open seating tickets Melody sold.
   b. The amount of money Melody collected from reserved seating tickets.
   c. The amount of money Melody collected from open seating tickets.
   d. The total amount of money Melody collected.

Solve Word Problems with a System of Equations

71. The chemistry department has 80 students, of whom 35% are women. The physics department has 60 students, of whom 15% are women.
   a. How many chemistry students are women? How many physics students are women?
   b. How many students are there in both departments? How many of them are women?
   c. What percent of the students in chemistry and physics are women?

<table>
<thead>
<tr>
<th>Number of Students (( W ))</th>
<th>Percent Women (( r ))</th>
<th>Number of Women (( P ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemistry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Physics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

72. Polls conducted by Senator Quagmire’s campaign manager show that he can win 60% of the rural vote in his state but only 45% of the urban vote. In the election, 1,200,000 citizens voted, \( x \) from rural areas and \( y \) from urban areas. Write expressions for:
   a. The number of rural votes for Quagmire
   b. The number of urban votes for Quagmire
   c. The total number of votes for Quagmire
   d. The percent all voters who voted for Quagmire

73. A pet store owner wants to mix a 12% saltwater solution and a 30% saltwater solution to obtain 45 liters of a 24% solution. How many liters of each ingredient does he need?

74. How many quarts of pure pigment must be mixed into a batch of paint that is 30% pigment to produce 24 quarts of paint that is 65% pigment?
75. A newspaper poll of 400 people stated that 58% were in favor of a recycling program. It also said that 50% of the men and 70% of the women polled favored the program. How many women were polled?

76. Bran flakes cost $1.40 per cup, and raisins cost $2.20 per cup. A 3-cup box of raisin bran costs $4.60. How many cups of raisins and how many cups of bran flakes are in the box?

77. Bonnie left Dallas and drove north at 40 miles per hour. Three hours later Clyde headed north form Dallas on the same road at 70 miles per hour until he caught up with Bonnie.
   a. Choose variables for Bonnie’s travel time and Clyde’s travel time. Complete the table.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonnie</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clyde</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Make a sketch showing Dallas, Bonnie, and Clyde. Label your sketch with the distance that each traveled.
   c. Use your table to write two equations about the problem.
   d. Solve your system. How long did Bonnie drive before Clyde caught up? How far had she driven?

78. A yacht leaves San Diego and heads south, traveling at 25 miles per hour. Six hours later a Coast Guard cutter leaves San Diego traveling at 40 miles per hour and pursues the yacht. How long will it take the cutter to catch the yacht? How far will they have traveled?

79. When Irma flies with the wind, she makes a 450-mile trip in 3 hours. The return trip against the wind takes 5 hours. Find the speed of the wind and the speed of the plane in still air.

**Write and Graph an Equation in Two Variables**

80. Boyer’s history book is 600 pages long, and he reads 20 pages per night. How many pages does he have left to read after \( t \) nights?

81. Kristi deposits $50 of her paycheck into savings and keeps 15% of the rest for spending money. If her paycheck is \( p \) dollars, how much spending money does she get?

82. The oven temperature started at 65° and is rising at 30° per minute. Write an expression for the oven temperature after \( t \) minutes.

83. Luisa’s parents have agreed to pay her tuition ($800 per year) plus half her annual living expenses while she is in school. Write an expression for the amount her parents will pay if Luisa’s annual expenses are \( a \) dollars.

84. Wrapping paper cost $1.50 per roll at the after-holidays sale. Let \( r \) stand for the number of rolls Elma buys, and \( p \) for the total price (before tax.) Write and graph an equation for \( p \) in terms of \( r \).
85. Uncle Ray’s diet allows him to eat a total of 1000 calories for lunch and dinner. Let \( l \) stand for the number of calories in Uncle Ray’s lunch, and \( d \) for the number of calories in his dinner. Write and graph an equation for \( d \) in terms of \( l \).

86. Greta’s math notebook has 100 pages, and she uses 6 pages per day on average for her notes and homework. How many pages, \( P \), will she have left after \( d \) days?

87. Asa has typed 220 words of his term paper, and is still typing at a rate of 20 words per minute. How many words, \( W \), will Asa have typed after \( m \) more minutes?

88. Delbert shares a house with four roommates. He pays $200 rent per month, plus his share of the utilities. If the utilities cost \( U \) dollars, how much money, \( M \), will Delbert owe?

89. For her roommate’s shower, Francine spent $50 on a gift plus her share of the cost of the party. If the party costs \( P \) dollars and 12 people are contributing (including Francine), what was the total amount, \( T \), that Francine spent?
a. Write an equation relating the variables.

b. Graph your equation.

c. Write and solve an equation to answer the question.

d. Illustrate the solution to part (c) on the graph.

90. Yakov has saved $300 and plans to add $20 per week to his savings. Write and graph an equation for Yakov’s savings, $S$, after $w$ weeks. How long will it take Yakov’s savings to grow to $560? 

91. Heidi planted a live 6-foot Christmas tree, and it is growing 4.5 inches per year. Write and graph an equation for the height, $h$, of the tree after $y$ years. (Hint: How many inches are in 6 feet?) How long will it take the tree to grow to a height of 9 feet?

92. The water in Silver Pond is 10 feet deep, but the water level is dropping at a rate of $\frac{1}{2}$ inch per week.

a. Write an equation for the depth, $d$, of the pond after $w$ weeks.

b. Complete the table of values and graph your equation on the grid at right.

c. How long will it take until the depth of the pond is 8 feet? (Hint: Convert all units to inches.)

Interpret Intercepts and Slope

a. Find the intercepts of each linear equation.

b. Use the intercept method to graph the line.

c. Explain what the intercepts mean in terms of the problem situation.

93. The amount of home heating oil (in gallons) in the Olsons’ tank is given by the equation $G = 200 - 15w$, where $w$ is the number of weeks since they turned on the furnace.

94. Dana joined a savings plan some weeks ago. Her bank balance is growing each week according to the formula $B = 225 + 25w$, where $w = 0$ represents this week.
95. Greg is monitoring the growth of a new variety of string beans. The height of the vine each day is given in inches by $h = 18 + 3d$, where $d = 0$ represents today.

![](image1)

96. Delbert bought some equipment and went into the dog-grooming business. His profit is increasing according to the equation $P = -600 + 40d$, where $d$ is the number of dogs he has groomed.

- a. Compute the slope of the graph, including units.
- b. Interpret the slope as a rate; what does it tell you about the problem?

97. Audrey can drive 150 miles on 6 gallons of gas, and 225 miles on 9 gallons of gas. Write an equation for the distance, $d$, that Audrey can drive on $g$ gallons of gas.

![](image2)

98. Jeremy can type 120 words in 4 minutes, and 450 words in 15 minutes. Write an equation for the number of words $w$ Jeremy can type in $t$ minutes.

![](image3)

II Simplify Expressions

Order of Operations
- Simplify by following the order of operations.
1. $15 - \frac{3}{4}(16)$
2. $\frac{1}{3} + \frac{2}{3} \cdot 2$
3. $20 - 3(6 - 4)$
4. $\frac{1}{3} \cdot 12 - 3 \left(\frac{5}{6}\right)$
5. $2 + 3 \cdot 8 - 6 + 3$
6. $24 + 6 + 2 \cdot 8 \div 4$
7. $\frac{3(8)}{12} - \frac{6 + 4}{5}$
8. $\frac{7 \cdot 5 - 2 \cdot 4}{5 - 2}$
9. $\frac{8}{18 - [8 - 12 - (-4)]}$
10. $5(-4) - 3(-6)$
11. $(4 - 3)(4 + 3)$
12. $-3(8) - 6(-2) - 5(2)$
13. $-6 - 3(-4 - 2)$
14. $\frac{4 - 8}{-3} - \frac{8 - 12}{-3}$
15. $4 - 2(-5)$
Evaluation

16. Evaluate for the given values.

\[8(s - t)\] for \(s = 27\) and \(t = 15\)

17. \[4a + 3b\] for \(a = 8\) and \(b = 7\)

18. \[\frac{w + 2z}{w}\] for \(w = 9\) and \(z = 3\)

19. \[\frac{a - b}{b}\] for \(a = 8\) and \(b = 6\)

20. \(v + gt\) for \(v = \frac{9}{4}\), \(g = 16\), and \(t = \frac{3}{2}\)

21. \[\frac{h(b + c)}{2}\] for \(h = 5.7\), \(b = 8.1\), and \(c = 2.9\)

22. \[15 - x - y\] for \(x = -6\), \(y = 8\)

23. \(p - (4 - m)\) for \(p = -2\), \(m = -6\)

24. \[12x - 3xy\] for \(x = -3\), \(y = 2\)

25. \(ab(6 - 4a)\) for \(a = -6\), \(b = -2\)

26. \[\frac{y - 3}{x - 4}\] for \(x = -9\), \(y = 2\)

27. \[\frac{56 - h}{t}\] for \(h = 200\), \(t = 3\)

28. \[\frac{5}{9}(F - 32)\] for \(F = -13\)

29. \[\frac{1 + e}{1 - e}\] for \(e = 0.4\)

30. \[\frac{1}{2}(t - 1)\] for \(t = \frac{2}{3}\)

31. \[\frac{1}{3}(h + 1)(h - 1)\] for \(h = -\frac{2}{5}\)

32. The area of a trapezoid with bases \(B\) and \(b\) and height \(h\) is given by \(A = \frac{1}{2}(B + b)h\). Find the area of a trapezoid whose bases are 9 centimeters and 7 centimeters and whose height is 3 centimeters.

33. The surface area of an open box with a square base is given by \(A = s(4h + s)\), where \(s\) stands for the length of one side of the base and \(h\) stands for the height of the box. Find the surface area of such a box whose base length is 2.4 feet and whose height is 1.6 feet.

34. Evaluate for \(x = -2\).

a. \(5x^3\)

b. \(5x^2\)

c. \(5 - x^2\)

d. \(5 - x^3\)

35. Evaluate for \(w = -9\).

a. \((2w)^2\)

b. \(36 - (2w)^2\)

c. \(-2(4 - w)^2\)

d. \(2(4 - w)^3\)

36. Evaluate for \(a = -3\), \(b = -4\).

a. \(ab^5\)

b. \(a - b^3\)

c. \((a - b^2)^2\)

d. \(ab(a^2 - b^2)\)

Laws of Exponents

Simplify by using the laws of exponents.

37a. \(x^3 \cdot x^5\)

b. \(5^6 \cdot 5^8\)

c. \(b^3(b)(b^5)\)

38a. \(a^6\)

b. \(3^9 \cdot 3^3\)

c. \(\frac{u^4}{u^8}\)

39a. \((t^3)^5\)

b. \((3^4)^3\)

c. \((3x)^3\)

40a. \((-2ab)^5\)

b. \((4 \cdot 100)^2\)

c. \((1.93 \times 10^3)^4\)

41a. \(\left(\frac{w}{2}\right)^6\)

b. \(\left(-\frac{m}{p}\right)^3\)

c. \(\frac{2^4}{5}\)

42a. \(x^3 \cdot x^6\)

b. \((x^3)^6\)

c. \(\frac{x^3}{x^6}\)

d. \(\frac{x^6}{x^5}\)

43a. \(5 \cdot 5^{15}\)

b. \((5^{15})^{15}\)

c. \(\frac{5}{5^{15}}\)

d. \(\frac{5^{15}}{5}\)

44. \((2p^3)^5\)

45. \(\left(\frac{-3}{q^4}\right)^5\)

46. \(\left(\frac{-2h^2}{m^3}\right)^4\)

47. \((x^2y)^3(xy^3)\)

48. \([ab^2(a^2b)^3]^3\)

49. \((-x)^3(-x^3)^3\)
Terms and Factors

- Simplify each pair of expressions.
  50. a. $x + x + x$
     b. $x \cdot x \cdot x$
  51. a. $5b + 5b$
     b. $5b \cdot 5b$
  52. a. $-3m - 3m$
     b. $(-3m)(-3m)$

- Simplify each expression if possible.
  53a. $5a^2 - 7a^2$
  53b. $3t - 2t^2$
  53c. $-m^2 - m^2$
  54a. $3k(4k)$
  54b. $3k + 4k$
  54c. $3k + 4k^2$
  55a. $3k(4k^2)$
  55b. $3k^2 + 4k^2$
  55c. $3k^2 - 4k^2$

- Simplify each pair of expressions as much as possible.
  56a. $4x^2 + 2x^4$
  56b. $4x^2(2x^4)$
  57a. $(-x)^3 - x^4$
  57b. $(-x)^3x^4$
  58a. $x^2(-3x^2)^3$
  58b. $x^2 - (-3x^2)^3$
  59a. $(3x^2)^4(2x^4)^2$
  59b. $(3x^2)^4 - (2x^4)^2$
  60a. $3x^2y - (3x)^2y$
  60b. $3xy^2 - (3xy)^2$
  61a. $6x^3 - 3x^6$
  61b. $6x^3(-3x^6)$

62. Decide which of the following products can be simplified.
   a. $2x^3(-3x^4)$
   b. $-x^4(-2x^2)$
   c. $-x^4 \cdot y^3$
   d. $-3x^5(3y^5)$

63. Decide which of the following sums can be simplified.
   a. $2x^3 - 3x^4$
   b. $-x^4 - 2x^2$
   c. $-x^4 + y^3$
   d. $-3x^5 + 3y^5$

64. For each pair of expressions, first find their product, then find their sum.
   a. $3a$, $8a$
   b. $-2b^3$, $5b^3$
   c. $4p^2q$, $-7p^2q$

Negative Exponents

- Write without using zero or negative exponents and simplify.
  65a. $5^{-2}$
  65b. $x^{-6}$
  65c. $(8x)^0$
  65d. $\left(\frac{3}{4}\right)^{-3}$
  66a. $\left(\frac{b}{3}\right)^{-4}$
  66b. $(2q)^{-5}$
  66c. $3 \cdot 4^{-3}$
  66d. $4x^{-2}$
  67a. $\frac{1}{6^{-3}}$
  67b. $\frac{3}{2^{-6}}$
  67c. $\frac{8x^3}{y^{-5}}$

- Write each expression using negative exponents.
  68a. $\frac{1}{2^3}$
  68b. $\frac{3}{5^2}$
  68c. $\frac{1}{27}$
  69a. $\frac{x}{625}$
  69b. $\frac{3}{z^2}$
  69c. $\left(\frac{z}{10}\right)^5$

- Simplify by using the laws of exponents.
  70a. $x^{-3} \cdot x^8$
  70b. $5^{-4} \cdot 5^{-3}$
  70c. $(3b^{-5})(5b^2)$
  71a. $c^{-7}$
  71b. $\frac{8b^{-4}}{c^{-4}}$
  71c. $6^9$
  72a. $\frac{(8-2)^5}{c^{-4}}$
  72b. $\frac{4b^{-8}}{(w^{-6})^{-3}}$
  72c. $(pq)^{-5}$
  73a. $\frac{(3x)^{-2}}{5(2r)^{-3}}$
  73b. $\frac{5(k^{-3})^4k^{-3}}{6k^{-5}}$
  73c. $(a^{-4}c)^{-3}$
  74. $(2u^{-2})^{-3}(u^{-4})^2$
  75. $\frac{5k^{-3}(k^4)^{-3}}{6k^{-5}}$
  76. $\left(\frac{2p^{-3}}{p^2}\right)^{-2}$

- Use scientific notation to compute.
  77. $(2,000,000)(0.00007)$
  78. $0.000036 ÷ 0.0009$
  79. $\frac{80,000,000,000}{20,000}$
III Polynomials

Operations on Polynomials

Multiply.
1. \( w^3(-8w^4) \)
2. \(-5s^2(2s^7t) \)
3. \(-6xy^2(-3xy^3)(-2xy) \)
4. \(-2b(6b - 2) \)
5. \(-4x^2(2x + 3y) \)
6. \((y^3 + 3y - 2)(2y) \)
7. \(-xy(2x^2 - xy + 3y^2) \)
8. \(2(3x - 1)(x - 3) \)
9. \(-3(x + 4)(x - 1) \)
10. \(-(4x + 3)(x - 2) \)
11. \((x + 3)(x - 3) \)
12. \((x + 5)(x - 5z) \)
13. \((2x - 3)(2x + 3) \)
14. \((w + 4)(w + 4) \)
15. \((m - 9)(m - 9) \)
16. \((2b + 5c)(2b + 5c) \)
17. \(-xy(x^2 + xy + y^2) \)
18. \((2a^2 - 6ab + b^2)(3b^3) \)
19. \(4a(a - 1)(a + 5) \)
20. \(-2w^3(3w + 2)^2 \)
21. \(s^2t^2(2s + t)(3s - t) \)
22. \((x - 2)(x^2 - 3x + 2) \)
23. \((3x + 2)(3x - 2)(x + 2) \)

Add or subtract the polynomials.
24. \((2y^3 - 4y^2 - y) + (6y^2 + 2y + 1) \)
25. \((2ab^2 + 6ab - a - 3b^2) - (2ab + a + 3b^2 + 2) \)

Simplify.
26. \(2a(x + 3) - 3a(x - 3) \)
27. \(2x(3 - x) + 2(x^2 + 1) - 2x \)
28. \(ax(x^2 + 2x - 3) - a(x^3 + 2x^2) \)
29. \(3ab^2(2 + 3a) - 2ab(3ab + 2b) \)

Divide.
30. \(\frac{8x^2y}{12x^3y^3} \)
31. \(\frac{-15x^3y^2}{-3x^2y^4} \)
32. \(\frac{6a^4 - a^2 - 2}{4a^2} \)

Factoring

Factor out the greatest common factor.
33. \(9x^2 - 12x^5 + 3x^3 \)
34. \(14x^3y - 35x^2y^2 + 21xy^3 \)
35. \(2x(x + 6) - 3(x + 6) \)
36. \(3x^2(2x + 3) - (2x + 3) \)

Factor out a negative monomial.
37. \(-4k^4 + 4k^2 - 2k \)
38. \(-3x^2 + 3xy - 3xy^2 \)
39. \(-8 - 4b \)

Factor completely.
40. \(5c^2 - 25c + 30 \)
41. \(4a^2b + 12ab - 7b \)
42. \(3p^2 - 30p^2 + 63p \)
43. \(18a^2b - 9ab - 27b \)
44. \(x^2 - 5xy + 6y^2 \)
45. \(4x^3 + 12x^2y + 8xy^2 \)
46. \(9a^3b + 9a^2b^2 - 18ab^3 \)
47. \(2t^2 - 5st - 3s^2 \)
48. \(4b^2y^2 + 5by + 1 \)
49. \(1 - 16v + 64v^2 \)
50. \(4p^2 - 12pq + 9q^2 \)
51. \(64y^4 - 49x^2 \)
52. \(1 - 100a^2 + 25 \)
53. \(a^4 + 10a^2 + 25 \)
54. \(x^6 - 64 \)
55. \(b^{10} - 12b^5 + 36 \)
56. \(16x^6 - 9y^4 \)
57. \(3a^2 - 75 \)
58. \(2a^3 - 12a^2 + 18a \)
59. \(8a^2b^4 - 18a^4b^2 \)
60. \(12h^2 + 3k^6 \)

IV Radicals

Order of Operations

Simplify each expression. Do not use a calculator!

1a. \(4 - 2\sqrt{64} \)
1b. \(4 - \sqrt{64} \)
1c. \(\sqrt{225} - \sqrt{49} \)
2a. \(\frac{36}{6 + \sqrt{36}} \)
2b. \(\sqrt{9} - 4(-18) \)
2c. \(\frac{\sqrt{4(50)} - 56}{16} \)
3a. \(\frac{5\sqrt{8} - \sqrt{64}}{8} \)
3b. \(3 + \sqrt[3]{729} \)
3c. \(8 - 2\sqrt[3]{-125} \)
Give a decimal approximation rounded to thousandths.

4a. \( \frac{-2}{3} \sqrt{21} \)  
   b. \( 5 - 3\sqrt{7} \)  
   c. \( \frac{6 + 9\sqrt{3}}{3} \)

5a. \( 2(\sqrt{10} - 5) + \sqrt{6} \)  
   b. \( 2 + 6\sqrt{-25} \)

Properties of Radicals

6. Use the definition of square root to simplify each expression.
   a. \( \sqrt{5}(\sqrt{5}) \)  
   b. \( \sqrt{x}(\sqrt{x}) \)  
   c. \( \frac{n}{\sqrt{n}} \)

   Simplify each square root
   7a. \( \sqrt{18} \)  
   b. \( -\sqrt{20} \)  
   c. \( \sqrt{80} \)
   8a. \( \sqrt{5^2} \)  
   b. \( \pm \sqrt{b^4} \)  
   c. \( \sqrt[3]{8a^3} \)
   9a. \( \sqrt{72m^6} \)  
   b. \( \sqrt{80p^7} \)  
   c. \( \sqrt[3]{\frac{32}{n^7}} \)

Operations on Radicals

10. \( \sqrt{7} - 3\sqrt{7} \)  
    11. \( 3\sqrt{5} - 3\sqrt{7} \)  
    12. \( 2\sqrt{6} - 9\sqrt{6} \)
    13. \( \sqrt{20} + \sqrt{45} - 2\sqrt{80} \)  
    14. \( \sqrt{18} - 5\sqrt{2} + 3\sqrt{12} \)

15. \( \sqrt{3} \sqrt{27} \)  
    16. \( \sqrt{2x}\sqrt{3x} \)  
    17. \( (3\sqrt{8a})(a\sqrt{18a}) \)
    18. \( \sqrt{3}(\sqrt{6} - 2) \)  
    19. \( \sqrt{5}(4 + 2\sqrt{15}) \)  
    20. \( (3 + \sqrt{2})(1 - \sqrt{2}) \)
    21. \( (5 + \sqrt{9})(3 + \sqrt{9}) \)  
    22. \( (4 - 3\sqrt{6})(4 + 3\sqrt{6}) \)  
    23. \( (2 + \sqrt{3})^2 \)
    24. \( (2\sqrt{n} - 1)^2 \)  
    25. \( (3\sqrt{5} + 2\sqrt{v})^2 \)  
    26. \( (2\sqrt{w} + \sqrt{5})(\sqrt{w} - 2\sqrt{5}) \)

10. \( \sqrt{7} - 3\sqrt{7} \)  
    11. \( 3\sqrt{5} - 3\sqrt{7} \)  
    12. \( 2\sqrt{6} - 9\sqrt{6} \)
    13. \( \sqrt{20} + \sqrt{45} - 2\sqrt{80} \)  
    14. \( \sqrt{18} - 5\sqrt{2} + 3\sqrt{12} \)

Multiply.

15. \( \sqrt{3} \sqrt{27} \)  
    16. \( \sqrt{2x}\sqrt{3x} \)  
    17. \( (3\sqrt{8a})(a\sqrt{18a}) \)
    18. \( \sqrt{3}(\sqrt{6} - 2) \)  
    19. \( \sqrt{5}(4 + 2\sqrt{15}) \)  
    20. \( (3 + \sqrt{2})(1 - \sqrt{2}) \)
    21. \( (5 + \sqrt{9})(3 + \sqrt{9}) \)  
    22. \( (4 - 3\sqrt{6})(4 + 3\sqrt{6}) \)  
    23. \( (2 + \sqrt{3})^2 \)
    24. \( (2\sqrt{n} - 1)^2 \)  
    25. \( (3\sqrt{5} + 2\sqrt{v})^2 \)  
    26. \( (2\sqrt{w} + \sqrt{5})(\sqrt{w} - 2\sqrt{5}) \)

Reduce if possible.

27. \( \frac{9 - 3\sqrt{5}}{3} \)  
    28. \( \frac{-8 + \sqrt{8}}{4} \)  
    29. \( \frac{6a - \sqrt{18}}{6a} \)

Write the expression as a single fraction in simplest form.

30. \( \frac{3\sqrt{5}}{4} - \frac{\sqrt{3}}{5} \)  
    31. \( \frac{3}{2a} + \frac{\sqrt{3}}{6a} \)  
    32. \( \frac{3\sqrt{3}}{2} + 3 \)
    33. \( \frac{3}{4} - 2\sqrt{y} \)  
    34. \( \frac{2\sqrt{y}}{9} + \frac{\sqrt{x}}{12} \)

Simplify the quotient.

35. \( \frac{\sqrt{48}}{\sqrt{3}} \)  
    36. \( \frac{\sqrt{75x^3}}{\sqrt{3x}} \)  
    37. \( \frac{\sqrt{48b}}{\sqrt{27b}} \)

Solve Equations

Solve and check.

39. \( \sqrt{x+4} = 5 \)  
    40. \( \sqrt{x-4} = 5 \)  
    41. \( 6 - \sqrt{x} = 8 \)
    42. \( 2 + 3\sqrt{x-1} = 8 \)  
    43. \( 2\sqrt{3x+1} - 3 = 5 \)  
    44. \( \sqrt{x} = 3 - 2x \)
    45. \( \sqrt{x+4} + 2 = x \)  
    46. \( \sqrt{x+7} = 2x + 4 \)  
    47. \( x + \sqrt{2x+3} - 7 = 9 \)
V Quadratic Equations

Solve a Quadratic Equation

☐ Solve by extraction of roots. Round your answers to three places if necessary.

1. \( w^2 - 125 = 0 \)
2. \( 98 = 2a^2 \)
3. \( 108 - 3b^2 = 0 \)
4. \( 0 = 24 - 4t^2 \)
5. \( 400 + \frac{k^2}{6} = 625 \)
6. \( 12 - 5v^2 = 2 \)
7. \( (3x - 2)^2 = 12 \)
8. \( 3(2x - 8)^2 = 60 \)
9. \( \frac{3}{4}(x - 6)^2 = 18 \)

☐ Solve.

10. \( 10x^2 - 15x = 0 \)
11. \( 20x^2 = x \)
12. \( 0 = 144x + 3x^2 \)
13. \( x^2 + 3x - 10 = 0 \)
14. \( t^2 + t = 42 \)
15. \( 2x^2 - 10x = 12 \)
16. \( 0 = n^2 - 14n + 49 \)
17. \( 5q^2 = 10q \)
18. \( x(x - 4) = 21 \)
19. \( (x - 2)(x + 1) = 4 \)
20. \( (x + 3)^2 = 2x + 14 \)

☐ Solve by using the quadratic formula.

21. \( 3s^2 + 2s = 2 \)
22. \( p^2 + 1 = 3p \)
23. \( -4z^2 + 2z + 1 = 0 \)
24. \( \frac{x^2}{6} + x = \frac{2}{3} \)
25. \( v^2 + 3v - 2 = 9v^2 - 12v + 5 \)
26. \( m^2 - 3m = \frac{1}{3}(m^2 - 1) \)
27. \( -0.2x^2 + 3.6x - 9 = 0 \)

28. The area of a computer circuit board must be 60 square centimeters. The length of the circuit board should be 2 centimeters shorter than twice its width. Find the dimensions of the circuit board.

29. Steve’s boat locker is 2 feet longer than twice its width. Find the dimensions of the locker if the 13-foot mast of Steve’s boat will just fit diagonally across the floor of the locker.

30. Francine throws a diamond engagement ring from the top of the First National Bank in Louisville. After \( t \) seconds, the height of the ring in feet is given by

\[ h = -16t^2 - 4t + 512 \]

a. When does the ring pass a balcony 400 feet above the ground?

b. How long does it take the ring to reach the ground?

31. The volume of a large aquarium at the zoo is 2160 cubic feet. The tank is 10 feet wide, and its length is 6 feet less than twice its height. Find the dimensions of the aquarium.

32. The perimeter of a rectangle is 46 centimeters and its diameter is 17 centimeters. Find the dimensions of the rectangle.

☐ Each figure is a right triangle. Find the unknown sides.

33. \( \triangle \):

- Hypotenuse: 17
- Leg: \( 2x - 1 \)

34. \( \triangle \):

- Hypotenuse: \( \sqrt{13} \)
- Leg: \( 2k - 5 \)
Graph a Parabola

Make a table of values and graph the parabola.

35. \( y = x^2 - 3 \)  
36. \( y = 9 - x^2 \)  
37. \( a. \ y = (x + 2)^2 \)  
38. \( y = -(x - 4)^2 \)  
39. \( y = x^2 + 2x - 8 \)  
40. \( y = -x^2 + 6x \)

41. The bridge over the Rushing River at Marionville is 48 feet high. Francine stands on the bridge and tosses a rock into the air off the edge of the bridge. The height of the rock above the water \( t \) seconds later is given in feet by

\[ h = 48 + 32t - 16t^2 \]

a. Complete the table of values.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

b. Sketch a graph of the equation on the grid.

c. Estimate the height of the rock after 1.75 seconds. Verify your answer algebraically.

d. When is the rock about 40 feet above the water?

e. Write an equation for the question in part (d).

f. How long is the rock more than 60 feet high?

g. After reaching its highest point, how long is the rock falling before it hits the water?

42. Mitra plans to build a rectangular pen for her rabbits in the back yard. She has 48 feet of wire fence. If she builds a pen of width \( w \) feet, then the area of the pen is given in square feet by

\[ A = 24w - w^2 \]

a. Complete the table of values.

<table>
<thead>
<tr>
<th>( w )</th>
<th>( A )</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
</tr>
<tr>
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<tr>
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<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

b. Sketch a graph of the equation on the grid.

c. If the width of the pen is 10 feet, what is its area?

d. If the area of the pen is 120 square feet, what is its width? (There are two answers.)

e. Write an equation for the question in part (d).

f. What is the largest area that Mitra can enclose with her fence?

g. What are the dimensions of the largest rabbit pen Mitra can build?
VI Algebraic Fractions

Write an Algebraic Fraction

- Write algebraic fractions to represent each of the following quantities.

1. Sharelle’s car still had 4 gallons in the gas tank when she filled up for $18.
   a. If the gas tank holds \( x \) gallons, what was the price per gallon of the gasoline?
   b. Evaluate your fraction for \( x = 14 \). What does your answer mean in the context of the problem?

2. Morgan drove across country in \( h \) hours, but he estimates that he spent 10 hours stopped for rest and meals.
   a. If he drove a total of 2800 miles, what was his average speed on the road?
   b. Evaluate your fraction for \( h = 50 \). What does your answer mean in the context of the problem?

3. The volume of a test tube is given by its height times the area of its cross-section. A test tube that holds 200 cubic centimeters is \( 2x - 1 \) centimeters long.
   a. What is the area of its cross-section?
   b. Evaluate your fraction for \( x = 13 \). What does your answer mean in the context of the problem?

4. It took Rashid \( 2x + 3 \) hours to type his autobiography.
   a. What fraction of the autobiography did Rashid type in 1 hour?
   b. What fraction of the autobiography did he type in 6 hours?

Operations on Fractions

- Reduce each fraction if possible, and select the correct response, a or b.

5. \( \frac{2x + 3}{2y} \)
   a. \( \frac{x + 3}{y} \)
   b. Cannot be reduced

6. \( \frac{3a + a^2}{3a} \)
   a. \( a^2 \)
   b. \( \frac{3 + a}{3} \)

7. \( \frac{y^2 - 1}{y - 1} \)
   a. \( y + 1 \)
   b. \( y \)

8. \( \frac{a^3}{a^2 - a^3} \)
   a. \( \frac{1}{a - 1} \)
   b. \( \frac{1}{a^3} \)

- Decide whether each fraction is equivalent to 1, to \(-1\), or cannot be reduced.

9. \( \frac{x + 4}{x - 4} \)
   a. \( \frac{t - 5w}{5w - t} \)
   b. \( \frac{x + 3z}{x + 3x} \)
   c. \( \frac{- (m - 1)}{1 - m} \)

- Decide which expressions are equivalent to the given fraction.

10. \( \frac{2}{3} \)
    a. \( \frac{2 + 4}{3 + 4} \)
    b. \( \frac{2 \cdot 4}{3 \cdot 4} \)
    c. \( \frac{22}{33} \)
    d. \( \frac{2 \div 4}{3 \div 4} \)

11. \( \frac{24}{32} \)
    a. \( \frac{24 - 12}{32 - 12} \)
    b. \( \frac{24 \div 6}{32 \div 8} \)
    c. \( \frac{2.4}{3.2} \)
    d. \( \frac{24^2}{32^2} \)

12. \( \frac{3}{5} \)
    a. \( \frac{3x}{5x} \)
    b. \( \frac{3 - x}{5 - x} \)
    c. \( \frac{30 + x}{x + 50} \)
    d. \( \frac{30x}{50x} \)

13. \( \frac{3a + 2}{3a - 2} \)
    a. \( \frac{3a - 2}{3a + 2} \)
    b. \( \frac{3a - 2}{2 - 3a} \)
    c. \( -1 \)
    d. \( \frac{2a + 3}{2a - 3} \)
Multiply.

14. \( \frac{2}{3x^3} \cdot \frac{9x^2}{4} \)  
15. \( \frac{-24a}{5b} \cdot \frac{15ab}{14} \)  
16. \( \frac{-v}{v+1} \cdot \frac{v}{v-1} \)  
17. \( \frac{3x-9}{2x-15} \cdot \frac{10x-5}{8x-4} \)  
18. \( \frac{5a+25}{5a} \cdot \frac{10a}{2a+10} \)  
19. \( \frac{4}{3v} \left( \frac{2}{3v} - \frac{6}{v^2} - \frac{3}{4v} \right) \)  
20. \( \frac{4V}{D} \cdot \frac{LR}{DV} \)  
21. \( \frac{2L}{c} \left( 1 + \frac{V^2}{c^2} \right) \)  
22. \( \frac{4\pi}{c^2} \left( \frac{c}{4\pi} - H + cM \right) \)

Divide.

23. \( \frac{a^4}{b^4} \div \frac{ab^2}{b^5} \)  
24. \( \frac{3xy}{4x^4} \div (-12y^2) \)  
25. \( y \div \frac{3x}{y^3} \)  
26. \( \frac{2x-2y}{5xy} \div \frac{4x-4y}{xy} \)  
27. \( \frac{a^2 + ab}{2a^2 - ab} \div \frac{ab^2 + b^3}{4a - 2b} \)

Add or subtract.

28. \( \frac{5}{2a} + \frac{3}{2a} \)  
29. \( \frac{3w}{w+5} - \frac{2}{w+5} \)  
30. \( \frac{m^2 + 1}{m-1} - \frac{2m}{m-1} \)  
31. \( \frac{3}{x} - \frac{4}{y} \)  
32. \( 1 - \frac{3}{a} \)  
33. \( \frac{u-v}{w} - \frac{w}{v} \)  
34. \( \frac{t+2}{t} - \frac{t+3}{2} \)  
35. \( \frac{3}{x} - \frac{2}{x+1} \)  
36. \( \frac{3}{n+3} + \frac{4n}{n-3} \)  
37. \( h - \frac{3}{h+2} \)  
38. \( \frac{v+1}{v} - \frac{1}{v-1} \)  
39. \( \frac{5}{2x} + \frac{3}{4x^2} \)  
40. \( \frac{5r-2}{2} - \frac{3r+1}{6r} \)  
41. \( \frac{3}{2a-b} + \frac{1}{8a-4b} \)  
42. \( \frac{3k}{3k-4} - \frac{5}{k+6} \)  
43. \( a + \frac{N-a^2}{2a} \)  
44. \( \frac{s+1}{t+1} - \frac{s}{t} \)  
45. \( \frac{-H}{RT} + \frac{S}{R} \)  
46. \( \frac{q}{4\pi r} + \frac{qa}{2\pi r^2} \)  
47. \( \frac{1}{LC} - \left( \frac{R}{2L} \right)^2 \)  
48. \( \frac{L}{c-V} + \frac{L}{c+V} \)

Write algebraic fractions in simplest form for each problem.

49. The dimensions of a rectangular rug are \( \frac{12}{x} \) feet and \( \frac{12}{x-2} \) feet.
   a. Write an expression for the area of the rug.
   b. Write an expression for the perimeter of the rug.

50. Colonial Airline has a commuter flight between Richmond and Washington, a distance of 100 miles. The plane flies at \( x \) miles per hour in still air. Today there is a steady wind from the north at 10 miles per hour.
   a. How long will the flight from Richmond to Washington take?
   b. How long will the flight from Washington to Richmond take?
   c. How long will a round trip take?
   d. Evaluate your fractions in parts (a)-(c) for \( x = 150 \).

51. Francine’s cocker spaniel eats a large bag of dog food in \( d \) days, and Delbert’s sheep dog takes \( 5 \) days less to eat the same size bag.
   a. What fraction of a bag of dog food does Francine’s cocker spaniel eat in one day?
   b. What fraction of a bag of dog food does Delbert’s sheep dog eat in one day?
   c. If Delbert and Francine get married, what fraction of a bag of dog food will their dogs eat in one day?
   d. If \( d = 25 \), how soon will Delbert and Francine have to buy more dog food?
Solve Equations

52.  \(2 + \frac{5}{2x} = \frac{3}{x} + \frac{3}{2}\)

53.  \(\frac{y - 2}{3y - \frac{1}{3}} = \frac{14}{3}\)

54.  \(\frac{4}{x - 3} = \frac{5}{2x + 3}\)

55.  \(\frac{4}{x - 1} - \frac{4}{x + 2} = \frac{3}{7}\)

56.  \(\frac{15x}{1 + x^2} = 6\)

57.  \(\frac{x + 4}{x^2 - 2x} = \frac{1}{4}\)

58. A small lake in a state park has become polluted by runoff from a factory upstream. The cost for removing \(p\) percent of the pollution from the lake is given, in thousands of dollars, by \(C = \frac{25p}{100 - p}\). How much of the pollution can be removed for $25,000?

59. During the baseball season so far this year, Pete got hits 44 times out of 164 times at bat:
   a. What is Pete’s batting average so far? (Batting average is the fraction of at-bats that resulted in hits.)
   b. If Pete gets hits on every one of his next \(x\) at-bats, write an expression for his new batting average.
   c. How many consecutive hits does Pete need to raise his batting average to 0.350?

60.  \(A = \frac{1}{2}(b + c)\) for \(b\)

61.  \(m = \frac{y - k}{x - h}\) for \(x\)

62.  \(\frac{1}{R} = \frac{1}{A} + \frac{1}{B}\) for \(A\)

63.  \(C = \frac{rR}{r + R}\) for \(R\)